



ชื่อ-นามสกุล

เลขประจำตัว

No. 1

แบบฝึกหัดเรื่อง ตรีโกณมิติ

1. จงหาค่าของ

1.1) $\cos\left(\frac{\pi}{12}\right) =$

$$\begin{aligned} \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) &= \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) \\ &= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{2}}{4} + \frac{\sqrt{2}\cdot\sqrt{3}}{4} \quad \text{หรือ} \quad \frac{\sqrt{2}+\sqrt{6}}{4} \end{aligned}$$

1.2) $\sin(75^\circ) =$

$$\begin{aligned} \sin(30^\circ + 45^\circ) &= \sin(30^\circ)\cos(45^\circ) + \cos(30^\circ)\sin(45^\circ) \\ &= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{2}}{4} + \frac{\sqrt{2}\cdot\sqrt{3}}{4} \quad \text{หรือ} \quad \frac{\sqrt{2}+\sqrt{6}}{4} \end{aligned}$$

1.3) $\tan(105^\circ) =$

$$\begin{aligned} \tan(45^\circ + 60^\circ) &= \frac{\tan(45^\circ) + \tan(60^\circ)}{1 - \tan(45^\circ)\tan(60^\circ)} \\ &= \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \quad \text{หรือ} \quad -2 - \sqrt{3} \end{aligned}$$

1.4) $\sec\left(\frac{13\pi}{12}\right) =$

$$\begin{aligned} \text{ทุก } \sec(A) &= \frac{1}{\cos(A)} \\ \cos\left(\frac{13\pi}{12}\right) &= \cos\left(\pi + \frac{\pi}{12}\right) = \cos\pi \cdot \cos\frac{\pi}{12} - \sin\pi \cdot \sin\frac{\pi}{12} \\ &= (-1) \cdot \cos\frac{\pi}{12} = (-1) \cdot \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = (-1) \cdot \frac{\sqrt{2}+\sqrt{6}}{4} \\ \sec\left(\frac{13\pi}{12}\right) &= \frac{-4}{\sqrt{2}+\sqrt{6}} = \sqrt{2}-\sqrt{6} \end{aligned}$$

1.5) $\cot(-285^\circ) = \frac{1}{\tan(-285^\circ)}$

$$\begin{aligned} \tan(-285^\circ) &= \tan(75^\circ + (-360^\circ)) = \frac{\tan(75^\circ) + \tan(-360^\circ)}{1 - \tan(75^\circ)\tan(-360^\circ)} \\ &= \tan(75^\circ) = \frac{\sin(75^\circ)}{\cos(75^\circ)} \\ &= \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}} = 2+\sqrt{3} \\ \text{ดังนั้น } \cot(-285^\circ) &= \frac{1}{2+\sqrt{3}} = 2-\sqrt{3} \end{aligned}$$

1.6) $\cos(345^\circ) =$

$$\begin{aligned} \cos(75^\circ + 270^\circ) &= \cos(75^\circ)\cos(270^\circ) - \sin(75^\circ)\sin(270^\circ) \\ &= -(-1)\sin(75^\circ) \\ &= \frac{\sqrt{2}+\sqrt{6}}{4} \end{aligned}$$

1.7) $\csc\left(-\frac{5\pi}{12}\right) = \frac{1}{\sin\left(-\frac{5\pi}{12}\right)}$

$$\begin{aligned} \sin\left(-\frac{5\pi}{12}\right) &= \sin\left(\frac{\pi}{12} - \frac{\pi}{2}\right) \\ &= \sin\frac{\pi}{12}\cos\frac{\pi}{2} - \cos\frac{\pi}{12}\sin\frac{\pi}{2} = -\cos\left(\frac{\pi}{12}\right) \\ &= -\frac{\sqrt{2}+\sqrt{6}}{4} \\ \text{ดังนั้น } \csc\left(-\frac{5\pi}{12}\right) &= \frac{4}{\sqrt{2}+\sqrt{6}} = \sqrt{2}-\sqrt{6} \end{aligned}$$

1.8) $\tan\left(\frac{17\pi}{12}\right) =$

$$\begin{aligned} \tan\left(\frac{17\pi}{12}\right) &= \tan\left(\frac{15\pi}{12} + \frac{2\pi}{12}\right) = \tan\left(\frac{5\pi}{4} + \frac{\pi}{6}\right) \\ &= \frac{\tan\frac{5\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\frac{5\pi}{4}\tan\frac{\pi}{6}} = \frac{1+\sqrt{3}}{-1+\sqrt{3}} \\ &= 2+\sqrt{3} \end{aligned}$$

2. จงหาค่าของ

$$2.1) \frac{\tan\left(\frac{\pi}{36}\right) + \tan\left(\frac{5\pi}{36}\right)}{1 - \tan\left(\frac{\pi}{36}\right)\tan\left(\frac{5\pi}{36}\right)} =$$

$$\tan\left(\frac{\pi}{36} + \frac{5\pi}{36}\right) = \tan\left(\frac{6\pi}{36}\right) = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$$

$$= \boxed{\frac{\sqrt{3}}{3}}$$

$$2.2) \cos(35^\circ)\cos(10^\circ) - \sin(35^\circ)\sin(10^\circ) =$$

$$\cos(35^\circ + 10^\circ) = \cos(45^\circ)$$

$$= \boxed{\frac{\sqrt{2}}{2}}$$

$$2.3) \frac{\tan(50^\circ) - \tan(20^\circ)}{1 + \tan(50^\circ)\tan(20^\circ)} =$$

$$\tan(50^\circ - 20^\circ) = \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$= \boxed{\frac{\sqrt{3}}{3}}$$

$$2.4) \sin(85^\circ)\cos(25^\circ) - \cos(85^\circ)\sin(25^\circ) =$$

$$\sin(85^\circ - 25^\circ) = \sin(60^\circ)$$

$$= \boxed{\frac{\sqrt{3}}{2}}$$

$$2.5) \cos\left(\frac{5\pi}{9}\right)\cos\left(\frac{\pi}{18}\right) + \sin\left(\frac{5\pi}{9}\right)\sin\left(\frac{\pi}{18}\right) =$$

$$\cos\left(\frac{5\pi}{9} - \frac{\pi}{18}\right) = \cos\left(\frac{\pi}{2}\right) = \boxed{0}$$

$$2.6) \frac{2\tan(15^\circ)}{1 - \tan(15^\circ)^2} =$$

$$\tan(2(15^\circ)) = \tan(30^\circ)$$

$$= \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$$

$$= \boxed{\frac{\sqrt{3}}{3}}$$

$$2.7) 2\cos\left(\frac{\pi}{8}\right)^2 - 1 =$$

$$\cos\left(2\left(\frac{\pi}{8}\right)\right) = \cos\left(\frac{\pi}{4}\right)$$

$$= \boxed{\frac{\sqrt{2}}{2}}$$

$$2.8) 1 - 2\sin(22.5^\circ)^2 =$$

$$\cos(2(22.5^\circ)) = \cos(45^\circ)$$

$$= \boxed{\frac{\sqrt{2}}{2}}$$

$$2.9) \cos(15^\circ)^2 - \sin(15^\circ)^2 =$$

$$\cos(2(15^\circ)) = \cos(30^\circ)$$

$$= \boxed{\frac{\sqrt{3}}{2}}$$

$$2.10) 2\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right) =$$

$$\sin\left(2\left(\frac{\pi}{12}\right)\right) = \sin\left(\frac{\pi}{6}\right)$$

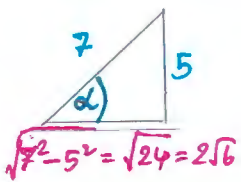
$$= \boxed{\frac{1}{2}}$$

3. กำหนด $A = \boxed{0}$, $B = \boxed{\frac{\pi}{2}}$, $C = \boxed{\frac{\pi}{2}}$, $D = \boxed{\pi}$

$\frac{\pi}{2} \leq \beta \leq \pi$ ใน Q_2

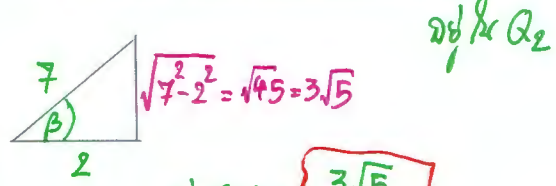
ถ้า $Con1 = \sin(\alpha) = \frac{5}{7}$, $A \leq \alpha \leq B$ และ $Con2 = \cos(\beta) = -\frac{2}{7}$, $C \leq \beta \leq D$ จงหา

3.1) $\cos(\alpha) =$ $\frac{2\sqrt{6}}{7}$ ใน Q_1



$\cos(\alpha) = \frac{2\sqrt{6}}{7}$

3.2) $\sin(\beta) =$



$\sin(\beta) = \frac{3\sqrt{5}}{7}$

3.3) $\tan(\beta) =$

$\frac{\pi}{2} \leq \beta \leq \pi$ ใน Q_2

$\tan(\beta) = \frac{-3\sqrt{5}}{2}$ ใน Q_2

3.4) $\sin(\alpha + \beta) =$

$\sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$
 $= \left(\frac{5}{7}\right)\left(-\frac{2}{7}\right) + \left(\frac{2\sqrt{6}}{7}\right)\left(\frac{3\sqrt{5}}{7}\right)$
 $= \frac{-10 + 6\sqrt{30}}{49}$

3.5) $\cos(\beta - \alpha) =$

$\cos(\beta - \alpha) = \cos(\beta)\cos(\alpha) + \sin(\beta)\sin(\alpha)$
 $= \left(-\frac{2}{7}\right)\left(\frac{2\sqrt{6}}{7}\right) + \left(\frac{3\sqrt{5}}{7}\right)\left(\frac{5}{7}\right)$
 $= \frac{-4\sqrt{6} + 15\sqrt{5}}{49}$

3.6) $\tan(\alpha - \beta) =$

$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$
 $= \frac{\frac{5}{2\sqrt{6}} - \left(-\frac{3\sqrt{5}}{2}\right)}{1 + \left(\frac{5}{2\sqrt{6}}\right)\left(-\frac{3\sqrt{5}}{2}\right)} = \frac{-2\sqrt{5}}{7} - \frac{10\sqrt{6}}{21}$

3.7) $\cos(2\beta) =$

$\cos(2\beta) = 2\cos^2(\beta) - 1$
 $= 2\left(-\frac{2}{7}\right)^2 - 1$
 $= \frac{-41}{49}$

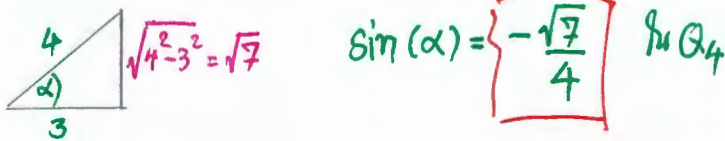
3.8) $\tan(2\alpha) =$

$\tan(2\alpha) = \frac{2\tan(\alpha)}{1 - \tan^2(\alpha)}$
 $= \frac{2\left(\frac{5}{2\sqrt{6}}\right)}{1 - \left(\frac{5}{2\sqrt{6}}\right)^2} = -20\sqrt{6}$

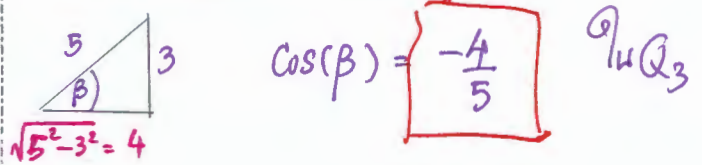
4. กำหนด $A = \frac{3\pi}{2}$, $B = 2\pi$, $C = \pi$, $D = \frac{3\pi}{2}$ $\pi \leq \beta \leq \frac{3\pi}{2}$ อยู่ใน Q_3

ถ้า $Con1 = \cos(\alpha) = \frac{3}{4}$, $A \leq \alpha \leq B$ และ $Con2 = \sin(\beta) = -\frac{3}{5}$, $C \leq \beta \leq D$ จงหา

4.1) $\sin(\alpha) =$



4.2) $\cos(\beta) =$



4.3) $\tan(\alpha) =$

$\tan(\alpha) = \frac{-\sqrt{7}}{3}$

4.4) $\sin(\alpha + \beta) =$

$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$
 $= \left(-\frac{\sqrt{7}}{4}\right)\left(-\frac{4}{5}\right) + \left(\frac{3}{4}\right)\left(-\frac{3}{5}\right)$
 $= \frac{\sqrt{7}}{5} - \frac{9}{20}$

4.5) $\cos(\alpha - \beta) =$

$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$
 $= \left(\frac{3}{4}\right)\left(-\frac{4}{5}\right) + \left(-\frac{\sqrt{7}}{4}\right)\left(-\frac{3}{5}\right)$
 $= -\frac{3}{5} + \frac{3\sqrt{7}}{20}$

4.6) $\tan(\beta - \alpha) =$

$\tan(\beta - \alpha) = \frac{\tan(\beta) - \tan(\alpha)}{1 + \tan(\beta)\tan(\alpha)}$
 $= \frac{\left(\frac{3}{4}\right) - \left(-\frac{\sqrt{7}}{3}\right)}{1 + \left(\frac{3}{4}\right)\left(-\frac{\sqrt{7}}{3}\right)} = \frac{9 + 4\sqrt{7}}{12 - 3\sqrt{7}} = \frac{64 + 25\sqrt{7}}{27}$

4.7) $\cos(2\beta) =$

$\cos(2\beta) = 2\cos^2(\beta) - 1$
 $= 2\left(-\frac{4}{5}\right)^2 - 1 = \frac{7}{25}$

4.8) $\tan(2\alpha) =$

$\tan(2\alpha) = \frac{2\tan(\alpha)}{1 - \tan^2(\alpha)} = \frac{2\left(-\frac{\sqrt{7}}{3}\right)}{1 - \left(-\frac{\sqrt{7}}{3}\right)^2}$
 $= -3\sqrt{7}$

5. ถ้า $Cond = \cos(44^\circ) = 0.719$

จงหา $Question = \cos(22^\circ)$

$\cos(22^\circ) = \cos\left(\frac{44^\circ}{2}\right) = \sqrt{\frac{1 + \cos(44^\circ)}{2}}$
 $= \sqrt{\frac{1 + 0.719}{2}} = 0.927$
 $= \sqrt{0.8595}$ หรือ

6. ถ้า $Cond = \cos(18^\circ) = 0.951$

จงหา $Question = \sin(9^\circ)$

$\sin(9^\circ) = \sin\left(\frac{18^\circ}{2}\right) = \sqrt{\frac{1 - \cos(18^\circ)}{2}}$
 $= \sqrt{\frac{1 - 0.951}{2}} = \sqrt{0.0245}$
 $= 0.156$

