

1. จงหาค่าของ
1.1) $\square$

$$
\begin{aligned}
\cos \left(\frac{\pi}{3}-\frac{\pi}{4}\right) & =\cos \left(\frac{\pi}{3}\right) \cos \left(\frac{\pi}{4}\right)+\sin \left(\frac{\pi}{3}\right) \sin \left(\frac{\pi}{4}\right) \\
& =\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)+\frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) \\
& =\frac{\sqrt{2}}{4}+\frac{\sqrt{2} \cdot \sqrt{3}}{4} \sqrt{40} \frac{\sqrt{2}+\sqrt{6}}{4}
\end{aligned}
$$

1.3)


$$
\begin{aligned}
\tan \left(45^{\circ}+60^{\circ}\right) & =\frac{\tan \left(45^{\circ}\right)+\tan \left(60^{\circ}\right)}{1-\tan \left(45^{\circ}\right) \cdot \tan \left(60^{\circ}\right)} \\
& =\frac{1+\sqrt{3}}{1-\sqrt{3}} \cdot 430-2-\sqrt{3}
\end{aligned}
$$



$$
\begin{aligned}
\tan \left(-285^{\circ}\right) & =\tan \left(75^{\circ}+\left(-360^{\circ}\right)\right)=\frac{\tan \left(75^{\circ}\right)+\tan \left(-360^{\circ}\right)}{1-\tan \left(75^{\circ}\right) \tan \left(-360^{\circ}\right)} \\
& =\tan \left(75^{\circ}\right)=\frac{\sin \left(75^{\circ}\right)}{\cos \left(75^{\circ}\right)} \\
& =\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}=2+\sqrt{3}
\end{aligned}
$$

ดัโu $\cot \left(-285^{\sqrt{6}-\sqrt{2}}=\frac{1}{2+\sqrt{3}}=2-\sqrt{3}\right.$
1.7) $\csc \left(-\frac{5 \pi}{12}\right)$

$$
\begin{aligned}
& \sin \left(\frac{-5 \pi}{12}\right)=\sin \left(\frac{\pi}{12}-\frac{\pi}{2}\right) \\
&=\sin \frac{\pi}{12} \cos \frac{\pi}{2}-\cos \frac{\pi}{12} \cdot \sin \frac{\pi}{2}=-\cos \left(\frac{\pi}{12}\right) \\
&=-\frac{(\sqrt{2}+\sqrt{6})}{4} \\
& \text { Dinv } \csc \left(-\frac{5 \pi}{12}\right)=-\frac{4}{\sqrt{2}+\sqrt{6}}=\sqrt{2}-\sqrt{6}
\end{aligned}
$$

$J=\frac{1}{\sin \left(-\frac{5 \pi}{12}\right)}$
1.8) $\tan \left(\frac{17 \pi}{12}\right)=$

$$
\begin{aligned}
\tan \left(\frac{17 \pi}{12}\right) & =\tan \left(\frac{15 \pi}{12}+\frac{2 \pi}{12}\right)=\tan \left(\frac{5 \pi}{4}+\frac{\pi}{6}\right) \\
& =\frac{\tan \frac{5 \pi}{4}+\tan \frac{\pi}{6}}{1-\tan \frac{5 \pi}{4} \cdot \tan \frac{\pi}{6}}=\frac{1+\sqrt{3}}{-1+\sqrt{3}} \\
& =2+\sqrt{3}
\end{aligned}
$$

2. จงหาค่าของ
2.1)


$$
\begin{aligned}
\tan \left(\frac{\pi}{36}+\frac{5 \pi}{36}\right) & =\tan \left(\frac{6 \pi}{36}\right)=\tan \left(\frac{\pi}{6}\right)=\frac{1}{\sqrt{3}} \\
& =\frac{\sqrt{3}}{3}
\end{aligned}
$$


2.3) $\square$

$$
=
$$

$$
\begin{aligned}
\tan \left(50^{\circ}-20^{\circ}\right) & =\tan \left(30^{\circ}\right)=\frac{1}{\sqrt{3}} \\
& =\frac{\sqrt{3}}{3}
\end{aligned}
$$

2.5) $\cos \left(\frac{5 \pi}{9}\right) \cos \left(\frac{\pi}{18}\right)+\sin \left(\frac{5 \pi}{9}\right) \sin \left(\frac{\pi}{18}\right)=$

$$
\cos \left(\frac{5 \pi}{9}-\frac{\pi}{18}\right)=\cos \left(\frac{\pi}{2}\right)=0
$$

2.7) $2 \cos \left(\frac{\pi}{8}\right)^{2}-1=$

$$
\begin{aligned}
\cos \left(2\left(\frac{\pi}{8}\right)\right) & =\cos \left(\frac{\pi}{4}\right) \\
& =\frac{\sqrt{2}}{2}
\end{aligned}
$$

2.9) $\square$

$$
\begin{aligned}
\cos \left(2\left(15^{\circ}\right)\right) & =\cos \left(30^{\circ}\right) \\
& =\frac{\sqrt{3}}{2}
\end{aligned}
$$

2.2) $\cos \left(35^{\circ}\right) \cos \left(10^{\circ}\right)-\sin \left(35^{\circ}\right) \sin \left(t^{\circ}\right)=$

$$
\begin{aligned}
\cos \left(35^{\circ}+10^{\circ}\right) & =\cos \left(45^{\circ}\right) \\
& =\frac{\sqrt{2}}{2}
\end{aligned}
$$

2.4) $\sin \left(85^{\circ}\right) \cos \left(25^{\circ}\right)-\cos \left(85^{\circ}\right) \sin \left(25^{\circ}\right)=$

$$
\begin{aligned}
\sin \left(85^{\circ}-25^{\circ}\right) & =\sin \left(60^{\circ}\right) \\
& =\frac{\sqrt{3}}{2}
\end{aligned}
$$

2.6) $\frac{2 \tan \left(15^{\circ}\right)}{1-\tan \left(15^{\circ}\right)^{2}}=$

$$
\begin{aligned}
\tan \left(2\left(15^{\circ}\right)\right) & =\tan \left(30^{\circ}\right) \\
& =\tan \left(\frac{\pi}{6}\right)=\frac{1}{\sqrt{3}} \\
& =\frac{\sqrt{3}}{3}
\end{aligned}
$$

2.8)

$$
\begin{aligned}
& 1-2 \sin (22.5)^{2} \\
& s(2(22.5))=\cos \left(45^{\circ}\right) \\
&=\frac{\sqrt{2}}{2}
\end{aligned}
$$

2.10) $2 \sin \left(\frac{\pi}{12}\right) \cos \left(\frac{\pi}{12}\right)=$

$$
\begin{aligned}
\sin \left(2\left(\frac{\pi}{12}\right)\right) & =\sin \left(\frac{\pi}{6}\right) \\
& =\frac{1}{2}
\end{aligned}
$$

3. กำหนด $A=$ $\square$

$$
B=
$$ $C=\frac{\pi}{2}$ , $D=\pi$ $\frac{\pi}{2} \leq \beta \leq \pi / 4 Q_{2}$ ถ้า $\operatorname{con} 1=\sin (\alpha)=\frac{5}{7}, \begin{aligned} & 0 \leq \alpha \leq B \\ & 0 \leq \alpha \leq \frac{\pi}{2}\end{aligned}$ และ $\operatorname{con} 2=\cos (\beta)=\frac{-2}{7}, C \leq \beta \leq D$ จงหา

3.1) $\square$ $\cos (\alpha)$ $=Q_{u} Q_{1}$

$\sqrt{7^{2}-5^{2}}=\sqrt{24}=2 \sqrt{6}$
3.3) $\tan (\beta)$

$$
\frac{\pi}{2} \leqslant \beta \leqslant \pi \text { \& } \theta_{2}
$$

$$
\tan (\beta)=-\frac{3 \sqrt{5}}{2} \quad \& Q_{2}
$$

3.5)


$$
\begin{aligned}
\cos (\beta-\alpha) & =\cos (\beta) \cos (\alpha)+\sin (\beta) \sin (\alpha) \\
& =\left(-\frac{2}{7}\right)\left(\frac{2 \sqrt{6}}{7}\right)+\left(\frac{3 \sqrt{5}}{7}\right) \cdot\left(\frac{5}{7}\right) \\
& =\frac{-4 \sqrt{6}+15 \sqrt{5}}{49}
\end{aligned}
$$

3.7)


$$
\begin{aligned}
\cos (2 \beta) & =2 \cos ^{2}(\beta)-1 \\
& =2\left(-\frac{2}{7}\right)^{2}-1 \\
& =\frac{-41}{49}
\end{aligned}
$$

3.2) $\square$
$\sin (\beta)$ $=$ osk $Q_{2}$
(3)
2
$7^{2}-2^{2}$
$=\sqrt{45}=3 \sqrt{5}$

$$
\sin (\beta)=\frac{3 \sqrt{5}}{7}
$$

3.4) $\square$
$\sin (\alpha+\beta)$

$$
\sin (\alpha) \cos (\beta)+\cos (\alpha) \cdot \sin (\beta)
$$

$$
=\left(\frac{5}{7}\right)\left(-\frac{2}{7}\right)+\left(\frac{2 \sqrt{6}}{7}\right)\left(\frac{3 \sqrt{5}}{7}\right)
$$

$$
=\frac{-10+6 \sqrt{30}}{49}
$$

3.6)

$$
\begin{aligned}
& \tan (\alpha-\beta) \\
& (\alpha-\beta)=\frac{\tan (\alpha)-\tan (\beta)}{1+\tan (\alpha) \cdot \tan (\beta)}
\end{aligned}
$$

$$
=\frac{\frac{5}{2 \sqrt{6}}-\left(-\frac{3 \sqrt{5}}{2}\right)}{1+\left(\frac{5}{2 \sqrt{6}}\right)\left(-\frac{3 \sqrt{5}}{2}\right)}=\frac{-2 \sqrt{5}}{7}-\frac{10 \sqrt{6}}{21}
$$

3.8)
$\tan (2 \alpha)=$

$$
\begin{aligned}
\tan (2 \alpha) & =\frac{2 \tan (\alpha)}{1-\tan ^{2}(\alpha)} \\
& =\frac{2\left(\frac{5}{2 \sqrt{6}}\right)}{1-\left(\frac{5}{2 \sqrt{6}}\right)^{2}}=-20 \sqrt{6}
\end{aligned}
$$

4. กำหนด $A=\frac{3 \pi}{2}, B=2 \pi, C=\pi, D=\frac{3 \pi}{2} \quad \pi \leqslant \beta \leqslant \frac{3 \pi}{2}$ of $Q_{3}$ ถ้า $\operatorname{Con} 1=\cos (\alpha)=\frac{3}{4}, \quad A \leq \alpha \leq B$ และ $\operatorname{con} 2=\sin (\beta)=\frac{-3}{5}, C \leq \beta \leq 2 \pi \quad Q_{4}$.,$C \leq$ จงา
4.1)

4.2)
$\cos (\beta)$

4.4) $\sin (\alpha+\beta)=$

$$
\begin{aligned}
\sin (\alpha+\beta) & =\sin (\alpha) \cos (\beta)+\cos (\alpha) \sin (\beta) \\
& =\left(-\frac{\sqrt{7}}{4}\right)\left(-\frac{4}{5}\right)+\left(\frac{3}{4}\right)\left(-\frac{3}{5}\right) \\
& =\frac{\sqrt{7}-\frac{9}{20}}{}
\end{aligned}
$$

4.3) $\tan (\alpha)=$

$$
\tan (\alpha)=-\frac{\sqrt{7}}{3}
$$

4.5)


$$
\begin{aligned}
\cos (\alpha-\beta) & =\cos (\alpha) \cos (\beta)+\sin (\alpha) \sin (\beta) \\
& =\left(\frac{3}{4}\right) \cdot\left(-\frac{4}{5}\right)+\left(-\frac{\sqrt{7}}{4}\right)\left(-\frac{3}{5}\right) \\
& =-\frac{3}{5}+\frac{3 \sqrt{7}}{20}
\end{aligned}
$$

4.7)


$$
\begin{aligned}
\cos (2 \beta) & =2 \cos ^{2}(\beta)-1 \\
& =2\left(-\frac{4}{5}\right)^{2}-1=\frac{7}{25}
\end{aligned}
$$

4.6) $\square$

$$
\begin{aligned}
\tan (\beta-\alpha) & =\frac{\tan (\beta)-\tan (\alpha)}{1+\tan (\beta) \tan (\alpha)} \\
& =\frac{\left(\frac{3}{4}\right)-\left(-\frac{\sqrt{7}}{3}\right)}{1+\left(\frac{3}{4}\right)(-\sqrt{3})}=\frac{9+4 \sqrt{7}}{12-3 \sqrt{7}}=\frac{64}{27}+\frac{25 \sqrt{7}}{27}
\end{aligned}
$$

4.8) $\tan (2 \alpha)=$

$$
\begin{aligned}
\tan (2 \alpha) & =\frac{2 \tan (\alpha)}{1-\tan ^{2}(\alpha)}=\frac{2\left(-\frac{\sqrt{7}}{3}\right)}{1-\left(-\frac{\sqrt{7}}{3}\right)^{2}} \\
& =-3 \sqrt{7}
\end{aligned}
$$

5. ถ้า $\operatorname{Cond}=\cos \left(44^{\circ}\right)=0.719$

$$
\text { จงหา Question }=\cos \left(22^{\circ}\right)
$$

$$
\begin{aligned}
\cos \left(22^{\circ}\right)=\cos \left(\frac{44^{\circ}}{2}\right) & =\sqrt{\frac{1+\cos \left(44^{\circ}\right)}{2}} \\
& =\sqrt{\frac{1+0.719}{2}}=0.927 \\
& =\sqrt{0.8595}
\end{aligned}
$$

$$
\begin{aligned}
\text { จงиา Question } & =\sin \left(9^{\circ}\right) \\
\sin \left(9^{\circ}\right)=\sin \left(\frac{18}{2}\right) & =\sqrt{\frac{1-\cos \left(18^{\circ}\right)}{2}} \\
& =\sqrt{\frac{1-0.951}{2}}=\sqrt{0.0245} \\
& =0.156
\end{aligned}
$$

$$
\begin{aligned}
& \text { No5 } \left.=\left[\text { Cond }=\left(\operatorname{Cos}\left(44^{\circ}\right)=0.719\right), \text { Question }=\operatorname{Cos}\left(22^{\circ}\right)\right], \quad, \begin{array}{c}
M \\
U \\
T
\end{array}\right] \\
& \text { No6 }=\left[\text { Cond }=\left(\operatorname{Cos}\left(18^{\circ}\right)=0.951\right) \text {, Question }=\operatorname{Sin}\left(9^{\circ}\right)\right], \quad,\left[\begin{array}{c}
M \\
U \\
T
\end{array}\right] \\
& \text { x [Page = 0001] xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx }
\end{aligned}
$$

$$
\begin{aligned}
& \text { Ans } l=\left[\begin{array}{cc}
.1=\left(\operatorname{Cos}\left(\frac{\pi}{12}\right)=\frac{\sqrt{2}}{4}+\frac{\sqrt{2} \sqrt{3}}{4}\right) & .2=\left(\operatorname{Sin}\left(75^{\circ}\right)=\frac{\sqrt{2}}{4}+\frac{\sqrt{2} \sqrt{3}}{4}\right) \\
.3=\left(\operatorname{Tan}\left(105^{\circ}\right)=-2-\sqrt{3}\right) & .4=\left(\operatorname{Sec}\left(\frac{13 \pi}{12}\right)=-\sqrt{2} \sqrt{3}+\sqrt{2}\right) \\
.5=\left(\operatorname{Cot}\left((-285)^{\circ}\right)=2-\sqrt{3}\right) & .6=\left(\operatorname{Cos}\left(345^{\circ}\right)=\frac{\sqrt{2}}{4}+\frac{\sqrt{2} \sqrt{3}}{4}\right) \\
.7=\left(\operatorname{Csc}\left(-\frac{5 \pi}{12}\right)=-\sqrt{2} \sqrt{3}+\sqrt{2}\right) & .8=\left(\operatorname{Tan}\left(\frac{17 \pi}{12}\right)=2+\sqrt{3}\right)
\end{array}\right],\left[\begin{array}{c}
\frac{j}{:} \\
M \\
a \\
t \\
h \\
@ \\
M \\
U \\
T \\
\frac{j}{:}
\end{array}\right]
\end{aligned}
$$

$$
\begin{aligned}
& \text { Ans } 3=\left[\begin{array}{cc}
.1=\left(\operatorname{Cos}(\alpha)=\frac{2 \sqrt{6}}{7}\right) & .2=\left(\operatorname{Sin}(\beta)=\frac{3 \sqrt{5}}{7}\right) \\
.3=\left(\operatorname{Tan}(\beta)=-\frac{3 \sqrt{5}}{2}\right) & .4=\left(\operatorname{Sin}(\alpha+\beta)=-\frac{10}{49}+\frac{6 \sqrt{6} \sqrt{5}}{49}\right) \\
.5=\left(\operatorname{Cos}(\beta-\alpha)=-\frac{4 \sqrt{6}}{49}+\frac{15 \sqrt{5}}{49}\right) & .6=\left(\operatorname{Tan}(\alpha-\beta)=-\frac{10 \sqrt{6}}{21}-\frac{2 \sqrt{5}}{7}\right) \\
.7=\left(\operatorname{Cos}(2 \beta)=\frac{-41}{49}\right) & .8=(\operatorname{Tan}(2 \alpha)=-20 \sqrt{6})
\end{array}\right], A n s 4=\left[\begin{array}{cc}
.1=\left(\operatorname{Sin}(\alpha)=-\frac{\sqrt{7}}{4}\right) & .2=\left(\operatorname{Cos}(\beta)=\frac{-4}{5}\right) \\
.3=\left(\operatorname{Tan}(\alpha)=-\frac{\sqrt{7}}{3}\right) & .4=\left(\operatorname{Sin}(\alpha+\beta)=\frac{\sqrt{7}}{5}-\frac{9}{20}\right) \\
.5=\left(\operatorname{Cos}(\alpha-\beta)=-\frac{3}{5}+\frac{3 \sqrt{7}}{20}\right) & .6=\left(\operatorname{Tan}(\beta-\alpha)=\frac{64}{27}+\frac{25 \sqrt{7}}{27}\right) \\
.7=\left(\operatorname{Cos}(2 \beta)=\frac{7}{25}\right) & .8=(\operatorname{Tan}(2 \alpha)=-3 \sqrt{7})
\end{array}\right],
\end{aligned}
$$

> Ans $5=\left(\operatorname{Cos}\left(22^{\circ}\right)=(\operatorname{Sqrt}(0.8595)=0.927)\right)$,
> ,$\left[\begin{array}{l}M \\ U \\ T\end{array}\right]$
> Ans $6=\left(\operatorname{Sin}\left(9^{\circ}\right)=(\operatorname{Sqrt}(0.0245)=0.156)\right), \quad,\left[\begin{array}{c}M \\ U \\ T\end{array}\right]$

