



แบบฝึกหัดเรื่อง ตรีโกณมิติ

ชื่อ-นามสกุล

เลขประจำตัว No. 3

1. จงหาค่าของ

1.1) $\cos\left(\frac{7\pi}{12}\right) = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

จาก $\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$
 $= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$
 $= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$
 $= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

1.2) $\sin(15^\circ) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

จาก $\sin(45^\circ - 30^\circ)$
 $= \sin(45^\circ)\cos(30^\circ) - \cos(45^\circ)\sin(30^\circ)$
 $= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$
 $= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

1.3) $\tan(75^\circ) = 2 + \sqrt{3}$

$\tan(75^\circ) = \tan(45^\circ + 30^\circ)$
 $= \frac{\tan(45^\circ) + \tan(30^\circ)}{1 - \tan(45^\circ)\tan(30^\circ)}$
 $= \frac{1 + \frac{\sqrt{3}}{3}}{1 - 1 \cdot \frac{\sqrt{3}}{3}} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}} \cdot \frac{3 + \sqrt{3}}{3 + \sqrt{3}} = 2 + \sqrt{3}$

1.4) $\cot(-105^\circ) = \frac{-1 + \sqrt{3}}{1 + \sqrt{3}}$

$\cot(105^\circ) = \frac{\cos(60^\circ + 45^\circ)}{\sin(60^\circ + 45^\circ)}$
 $= \frac{\cos(60^\circ)\cos(45^\circ) - \sin(60^\circ)\sin(45^\circ)}{\sin(60^\circ)\cos(45^\circ) + \cos(60^\circ)\sin(45^\circ)}$
 $= \frac{\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$
 1.4: $\cot(-105^\circ) = -\cot(105^\circ) = \frac{-1 + \sqrt{3}}{1 + \sqrt{3}}$

1.5) $\csc\left(-\frac{19\pi}{12}\right) = -\sqrt{2} + \sqrt{6}$

$\csc\left(-\frac{19\pi}{12}\right) = \frac{1}{\sin\left(-\frac{19\pi}{12}\right)} = \frac{1}{\sin\left(\frac{\pi}{4} - \frac{11\pi}{6}\right)}$
 $\frac{1}{\sin\left(\frac{\pi}{4} - \frac{11\pi}{6}\right)} = \frac{1}{\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{11\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{11\pi}{6}\right)}$
 $= \frac{1}{\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \left(-\frac{1}{2}\right)} = \frac{4}{\sqrt{6} + \sqrt{2}} \cdot \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}} = -\sqrt{2} + \sqrt{6}$

1.6) $\sin\left(-\frac{23\pi}{12}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$\sin\left(-\frac{23\pi}{12}\right) = \sin\left(-\frac{2\pi}{3} - \frac{5\pi}{4}\right)$
 $= \sin\left(-\frac{2\pi}{3}\right)\cos\left(\frac{5\pi}{4}\right) - \cos\left(-\frac{2\pi}{3}\right)\sin\left(\frac{5\pi}{4}\right)$
 $= \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$
 $= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$
 1.6: $\sin(-\theta) = -\sin(\theta)$ 1.6: $\cos(-\theta) = \cos(\theta)$

1.7) $\cos(-165^\circ) = -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$\cos(-165^\circ) = \cos(15^\circ - 180^\circ)$
 $= \cos(15^\circ)\cos(180^\circ) + \sin(15^\circ)\sin(180^\circ)$
 $= (-1)\cos(45^\circ - 30^\circ)$
 $= (-1)[\cos(45^\circ)\cos(30^\circ) + \sin(45^\circ)\sin(30^\circ)]$
 $= (-1)\left[\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}\right]$
 $= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

1.8) $\tan(165^\circ) = -2 + \sqrt{3}$

$\tan(165^\circ) = \tan(180^\circ - 15^\circ) = \frac{\tan(180^\circ) - \tan(15^\circ)}{1 + \tan(180^\circ)\tan(15^\circ)}$
 $= -\tan(15^\circ)$
 $\tan(15^\circ) = \tan(45^\circ - 30^\circ)$
 $= \frac{\tan(45^\circ) - \tan(30^\circ)}{1 + \tan(45^\circ)\tan(30^\circ)} = \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 \cdot \frac{\sqrt{3}}{3}}$
 $= \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}} = 2 - \sqrt{3}$
 ดังนั้น $-\tan(15^\circ) = -2 + \sqrt{3}$

2. จงหาค่าของ

$$2.1) \cos\left(\frac{\pi}{18}\right)\cos\left(\frac{\pi}{9}\right) - \sin\left(\frac{\pi}{18}\right)\sin\left(\frac{\pi}{9}\right) = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \text{จาก } & \cos\left(\frac{\pi}{18}\right)\cos\left(\frac{\pi}{9}\right) - \sin\left(\frac{\pi}{18}\right)\sin\left(\frac{\pi}{9}\right) \\ &= \cos\left(\frac{\pi}{18} + \frac{\pi}{9}\right) \\ &= \cos\left(\frac{\pi}{6}\right) \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$2.2) 2\sin(15^\circ)\cos(15^\circ) =$$

$$\begin{aligned} \text{จาก } & 2\sin(15^\circ)\cos(15^\circ) \\ &= \sin(30^\circ) \\ &= \frac{1}{2} \end{aligned}$$

$$2.3) \sin(48^\circ)\cos(12^\circ) + \cos(48^\circ)\sin(12^\circ) = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \text{จาก } & \sin(48^\circ)\cos(12^\circ) + \cos(48^\circ)\sin(12^\circ) \\ &= \sin(48^\circ + 12^\circ) \\ &= \sin(60^\circ) \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$2.4) 2\cos^2(22.5^\circ) - 1 = \frac{\sqrt{2}}{2}$$

$$\begin{aligned} \text{จาก } & 2\cos^2(22.5^\circ) - 1 = \cos(45^\circ) \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

$$2.5) \frac{\tan(50^\circ) - \tan(20^\circ)}{1 + \tan(50^\circ)\tan(20^\circ)} = \frac{\sqrt{3}}{3}$$

$$\begin{aligned} \text{จาก } & \frac{\tan(50^\circ) - \tan(20^\circ)}{1 + \tan(50^\circ)\tan(20^\circ)} \\ &= \tan(50^\circ - 20^\circ) \\ &= \tan(30^\circ) \\ &= \frac{\sqrt{3}}{3} \end{aligned}$$

$$2.6) 1 - 2\sin^2\left(\frac{\pi}{8}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{aligned} \text{จาก } & 1 - 2\sin^2\left(\frac{\pi}{8}\right) = \cos\left(\frac{\pi}{4}\right) \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

$$2.7) \cos^2\left(\frac{\pi}{12}\right) - \sin^2\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \text{จาก } & \cos^2\left(\frac{\pi}{12}\right) - \sin^2\left(\frac{\pi}{12}\right) \\ &= \cos\left(\frac{\pi}{6}\right) \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$2.8) \sin\left(\frac{11\pi}{18}\right)\cos\left(\frac{\pi}{9}\right) - \cos\left(\frac{11\pi}{18}\right)\sin\left(\frac{\pi}{9}\right) = 1$$

$$\begin{aligned} \text{จาก } & \sin\left(\frac{11\pi}{18}\right)\cos\left(\frac{\pi}{9}\right) - \cos\left(\frac{11\pi}{18}\right)\sin\left(\frac{\pi}{9}\right) \\ &= \sin\left(\frac{11\pi}{18} - \frac{\pi}{9}\right) \\ &= \sin\left(\frac{\pi}{2}\right) \\ &= 1 \end{aligned}$$

$$2.9) \frac{2\tan(22.5^\circ)}{1 - \tan^2(22.5^\circ)} = 1$$

$$\begin{aligned} \text{จาก } & \frac{2\tan(22.5^\circ)}{1 - \tan^2(22.5^\circ)} = \tan(45^\circ) \\ &= 1 \end{aligned}$$

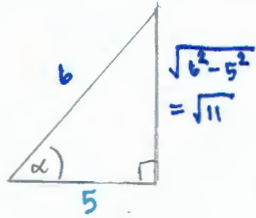
$$2.10) \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{12}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{aligned} \text{จาก } & \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{12}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{12}\right) \\ &= \cos\left(\frac{\pi}{3} - \frac{\pi}{12}\right) \\ &= \cos\left(\frac{\pi}{4}\right) \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

3. กำหนด $A = \boxed{0}$, $B = \boxed{\frac{\pi}{2}}$, $C = \boxed{\frac{3\pi}{2}}$, $D = \boxed{2\pi}$

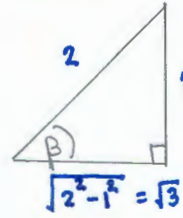
ถ้า $Con1 = \boxed{\cos(\alpha) = \frac{5}{6}}$, $A \leq \alpha \leq B$ และ $Con2 = \boxed{\sin(\beta) = -\frac{1}{2}}$, $C \leq \beta \leq D$ จงหา

3.1) $\boxed{\sin(\alpha) = \frac{\sqrt{11}}{6}}$



$0 \leq \alpha \leq \frac{\pi}{2}$
 $\sin(\alpha) = \frac{\sqrt{11}}{6}$

3.2) $\boxed{\cos(\beta) = \frac{\sqrt{3}}{2}}$



$\frac{3\pi}{2} \leq \beta \leq 2\pi$
 $\cos(\beta) = \frac{\sqrt{3}}{2}$

3.3) $\boxed{\tan(\beta) = -\frac{\sqrt{3}}{3}}$

$\frac{3\pi}{2} \leq \beta \leq 2\pi$
 $\tan(\beta) = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

3.4) $\boxed{\sin(\alpha - \beta) = \frac{\sqrt{3}\sqrt{11}}{12} + \frac{5}{12}}$

$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$
 $= \frac{\sqrt{11}}{6} \cdot \frac{\sqrt{3}}{2} - \frac{5}{6} \cdot (-\frac{1}{2})$
 $= \frac{\sqrt{3}\sqrt{11}}{12} + \frac{5}{12}$

3.5) $\boxed{\cos(\beta - \alpha) = \frac{5\sqrt{3}}{12} - \frac{\sqrt{11}}{12}}$

$\cos(\beta - \alpha) = \cos(\beta)\cos(\alpha) + \sin(\beta)\sin(\alpha)$
 $= \frac{\sqrt{3}}{2} \cdot \frac{5}{6} + (-\frac{1}{2}) \cdot \frac{\sqrt{11}}{6}$
 $= \frac{5\sqrt{3}}{12} - \frac{\sqrt{11}}{12}$

3.6) $\boxed{\tan(\alpha + \beta) = \frac{5\sqrt{11}}{16} - \frac{9\sqrt{3}}{16}}$

$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$
 $= \frac{\frac{\sqrt{11}}{5} + (-\frac{\sqrt{3}}{3})}{1 - \frac{\sqrt{11}}{5} \cdot (-\frac{\sqrt{3}}{3})}$
 $= \frac{3\sqrt{11} - 5\sqrt{3}}{15 + \sqrt{3}\sqrt{11}} \cdot \frac{15 - \sqrt{3}\sqrt{11}}{15 - \sqrt{3}\sqrt{11}} = \frac{5\sqrt{11}}{16} - \frac{9\sqrt{3}}{16}$

3.7) $\boxed{\sin(2\alpha) = \frac{5\sqrt{11}}{18}}$

$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$
 $= 2\left(\frac{\sqrt{11}}{6}\right)\left(\frac{5}{6}\right)$
 $= \frac{5\sqrt{11}}{18}$

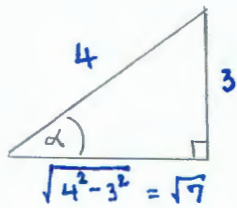
3.8) $\boxed{\tan(2\beta) = -\sqrt{3}}$

$\tan(2\beta) = \frac{2\tan(\beta)}{1 - \tan^2(\beta)}$
 $= \frac{2(-\frac{\sqrt{3}}{3})}{1 - (-\frac{\sqrt{3}}{3})^2}$
 $= \frac{-\frac{2\sqrt{3}}{3}}{\frac{2}{3}} = -\sqrt{3}$

4. กำหนด $A = \pi$, $B = \frac{3\pi}{2}$, $C = \frac{\pi}{2}$, $D = \pi$

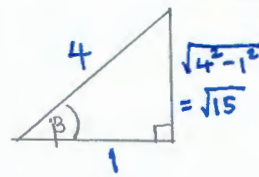
ถ้า $Con1 = \sin(\alpha) = -\frac{3}{4}$, $A \leq \alpha \leq B$ และ $Con2 = \cos(\beta) = -\frac{1}{4}$, $C \leq \beta \leq D$ จงหา

4.1) $\cos(\alpha) = -\frac{\sqrt{7}}{4}$



$\pi \leq \alpha \leq \frac{3\pi}{2}$
 $\cos(\alpha) = -\frac{\sqrt{7}}{4}$

4.2) $\sin(\beta) = \frac{\sqrt{15}}{4}$



$\frac{\pi}{2} \leq \beta \leq \pi$
 $\sin(\beta) = \frac{\sqrt{15}}{4}$

4.3) $\tan(\alpha) = \frac{3\sqrt{7}}{7}$

$\pi \leq \alpha \leq \frac{3\pi}{2}$

$\tan(\alpha) = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$

4.4) $\sin(\alpha - \beta) = \frac{3}{16} + \frac{\sqrt{7}\sqrt{15}}{16}$

$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$
 $= (-\frac{3}{4})(-\frac{1}{4}) - (-\frac{\sqrt{7}}{4})(\frac{\sqrt{15}}{4})$
 $= \frac{3}{16} + \frac{\sqrt{7}\sqrt{15}}{16}$

4.5) $\cos(\alpha + \beta) = \frac{\sqrt{7}}{16} + \frac{3\sqrt{15}}{16}$

$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$
 $= (-\frac{\sqrt{7}}{4})(-\frac{1}{4}) - (-\frac{3}{4})(\frac{\sqrt{15}}{4})$
 $= \frac{\sqrt{7}}{16} + \frac{3\sqrt{15}}{16}$

4.6) $\tan(\beta - \alpha) = \frac{\sqrt{15}}{8} + \frac{3\sqrt{7}}{8}$

$\tan(\beta - \alpha) = \frac{\tan(\beta) - \tan(\alpha)}{1 + \tan(\beta)\tan(\alpha)}$
 $= \frac{-\sqrt{15} - \frac{3}{\sqrt{7}}}{1 + (-\sqrt{15})(\frac{3}{\sqrt{7}})}$
 $= \frac{-\sqrt{15}\sqrt{7} - 3}{\sqrt{7} - 3\sqrt{15}} \cdot \frac{\sqrt{7} + 3\sqrt{15}}{\sqrt{7} + 3\sqrt{15}} = \frac{\sqrt{15}}{8} + \frac{3\sqrt{7}}{8}$

4.7) $\sin(2\alpha) = \frac{3\sqrt{7}}{8}$

$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$
 $= 2(-\frac{3}{4})(-\frac{\sqrt{7}}{4})$
 $= \frac{3\sqrt{7}}{8}$

4.8) $\tan(2\beta) = \frac{\sqrt{15}}{7}$

$\tan(2\beta) = \frac{2\tan(\beta)}{1 - \tan^2(\beta)}$
 $= \frac{2(-\sqrt{15})}{1 - (-\sqrt{15})^2} = \frac{\sqrt{15}}{7}$

5. ถ้า $Cond = \cos(86^\circ) = 0.070$

จงหา $Question = \sin(43^\circ)$

$\sin(43^\circ) = \sin(\frac{86^\circ}{2})$

$= \sqrt{\frac{1 - \cos(86^\circ)}{2}}$

$= \sqrt{0.465}$

มุม 43° อยู่บน
 47 องศา

6. ถ้า $Cond = \cos(20^\circ) = 0.940$

จงหา $Question = \cos(10^\circ)$

$\cos(10^\circ) = \cos(\frac{20^\circ}{2})$

$= \sqrt{\frac{1 + \cos(20^\circ)}{2}}$

$= \sqrt{0.97}$

มุม 10° อยู่บน
 97 องศา

