



แบบฝึกหัดเรื่อง ตรีโกณมิติ

ชื่อ-นามสกุล

เลขประจำตัว No. 3

1. จงหาค่าของ

1.1) $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

1.2) $\tan^{-1}(0) = 0$

1.3) $\sin^{-1}(1) = \frac{\pi}{2}$

1.4) $\cos^{-1}(0) = \frac{\pi}{2}$

1.5) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

1.6) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

1.7) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

1.8) $\sin^{-1}(1) = \frac{\pi}{2}$

1.9) $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

2. จงหาค่าของ

2.1) $\cos(\cos^{-1}(-\frac{\sqrt{3}}{2})) = -\frac{\sqrt{3}}{2}$
 $\cos(\cos^{-1}(-\frac{\sqrt{3}}{2})) = \cos(\frac{5\pi}{6})$
 $= -\frac{\sqrt{3}}{2}$

2.2) $\sin(\tan^{-1}(-1)) = -\frac{\sqrt{2}}{2}$
 $\sin(\tan^{-1}(-1)) = \sin(-\frac{\pi}{4})$
 $= -\frac{\sqrt{2}}{2}$

2.3) $\tan(\cos^{-1}(\frac{\sqrt{3}}{2})) = \frac{\sqrt{3}}{3}$
 $\tan(\cos^{-1}(\frac{\sqrt{3}}{2})) = \tan(\frac{\pi}{6})$
 $= \frac{\sqrt{3}}{3}$

2.4) $\cos(\tan^{-1}(1)) = \frac{\sqrt{2}}{2}$
 $\cos(\tan^{-1}(1)) = \cos(\frac{\pi}{4})$
 $= \frac{\sqrt{2}}{2}$

2.5) $\tan^{-1}(\tan(-\frac{\pi}{6})) = -\frac{\pi}{6}$
 $\tan^{-1}(\tan(-\frac{\pi}{6})) = \tan^{-1}(-\frac{\sqrt{3}}{3})$
 $= -\frac{\pi}{6}$

2.6) $\sin^{-1}(\cos(\frac{2\pi}{3})) = -\frac{\pi}{6}$
 $\sin^{-1}(\cos(\frac{2\pi}{3})) = \sin^{-1}(-\frac{1}{2})$
 $= -\frac{\pi}{6}$

2.7) $\sin^{-1}(\sin(-\frac{11\pi}{6})) = \frac{\pi}{6}$
 $\sin^{-1}(\sin(-\frac{11\pi}{6})) = \sin^{-1}(\sin(\frac{\pi}{6}))$
 $= \frac{\pi}{6}$

2.8) $\cos^{-1}(\cos(-\frac{\pi}{3})) = \frac{\pi}{3}$
 $\cos^{-1}(\cos(-\frac{\pi}{3})) = \cos^{-1}(\cos(\frac{\pi}{3}))$
 $= \frac{\pi}{3}$

2.9) $\tan^{-1}(\tan(-\frac{7\pi}{6})) = -\frac{\pi}{6}$
 $\tan^{-1}(\tan(-\frac{7\pi}{6})) = \tan^{-1}(\tan(-\frac{\pi}{6}))$
 $= -\frac{\pi}{6}$

2.10) $\sec(\tan^{-1}(-\frac{\sqrt{3}}{3})) = \frac{2\sqrt{3}}{3}$
 $\sec(\tan^{-1}(-\frac{\sqrt{3}}{3})) = \sec(-\frac{\pi}{6})$
 $= \frac{2\sqrt{3}}{3}$

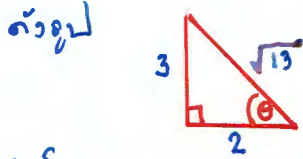
2.11) $\tan(\sin^{-1}(\cos(\frac{7\pi}{3}))) = \frac{3\sqrt{3}}{3}$
 $\tan(\sin^{-1}(\cos(\frac{7\pi}{3}))) = \tan(\sin^{-1}(\frac{1}{2}))$
 $= \tan(\frac{\pi}{6})$
 $= \frac{3\sqrt{3}}{3}$

2.12) $\sin^{-1}(\cos(\sin^{-1}(\frac{1}{2}))) = \frac{\pi}{3}$
 $\sin^{-1}(\cos(\sin^{-1}(\frac{1}{2}))) = \sin^{-1}(\cos(\frac{\pi}{6}))$
 $= \sin^{-1}(\frac{\sqrt{3}}{2})$
 $= \frac{\pi}{3}$

3. จงหาค่าของ

3.1) $\sin(\tan^{-1}(\frac{3}{2})) = \frac{3\sqrt{13}}{13}$

ให้ $\tan^{-1}(\frac{3}{2}) = \theta$
 จะได้ $\tan(\theta) = \frac{3}{2}$

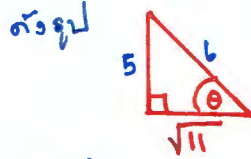


ดังนั้น

$\sin(\tan^{-1}(\frac{3}{2})) = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$

3.2) $\cos(\sin^{-1}(\frac{5}{6})) = \frac{\sqrt{11}}{6}$

ให้ $\sin^{-1}(\frac{5}{6}) = \theta$
 จะได้ $\sin(\theta) = \frac{5}{6}$

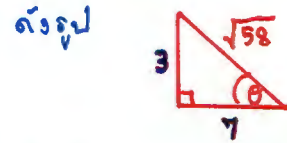


ดังนั้น

$\cos(\sin^{-1}(\frac{5}{6})) = \frac{\sqrt{11}}{6}$

3.3) $\csc(\tan^{-1}(-\frac{3}{7})) = -\frac{\sqrt{58}}{3}$

ให้ $\tan^{-1}(-\frac{3}{7}) = \theta$
 จะได้ $\tan(\theta) = -\frac{3}{7}$



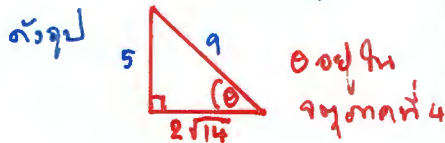
θ อยู่ใน
 จตุภาคที่ 4

ดังนั้น

$\csc(\tan^{-1}(-\frac{3}{7})) = -\frac{\sqrt{58}}{3}$

3.4) $\sec(\sin^{-1}(-\frac{5}{9})) = \frac{9\sqrt{14}}{24}$

ให้ $\sin^{-1}(-\frac{5}{9}) = \theta$
 จะได้ $\sin(\theta) = -\frac{5}{9}$

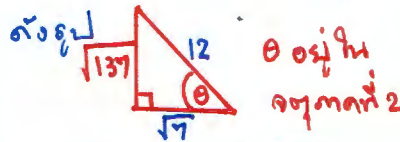


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 จตุภาคที่ 4

$\sec(\sin^{-1}(-\frac{5}{9})) = \frac{9}{2\sqrt{14}} = \frac{9\sqrt{14}}{28}$

3.5) $\tan(\cos^{-1}(-\frac{\sqrt{7}}{12})) = -\frac{\sqrt{137}\sqrt{7}}{7}$

ให้ $\cos^{-1}(-\frac{\sqrt{7}}{12}) = \theta$
 จะได้ $\cos(\theta) = -\frac{\sqrt{7}}{12}$

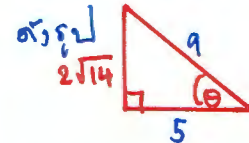


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 จตุภาคที่ 2

$\tan(\cos^{-1}(-\frac{\sqrt{7}}{12})) = -\frac{\sqrt{137}}{\sqrt{7}} = -\frac{\sqrt{137}\sqrt{7}}{7}$

3.6) $\sin(\cos^{-1}(\frac{5}{9})) = \frac{2\sqrt{14}}{9}$

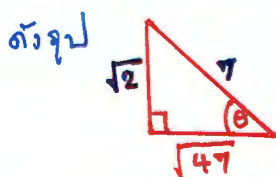
ให้ $\cos^{-1}(\frac{5}{9}) = \theta$
 จะได้ $\cos(\theta) = \frac{5}{9}$



$\sin(\cos^{-1}(\frac{5}{9})) = \frac{2\sqrt{14}}{9}$

3.7) $\tan(\sin^{-1}(\frac{\sqrt{2}}{7})) = \frac{\sqrt{2}\sqrt{47}}{47}$

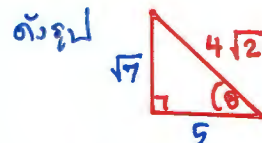
ให้ $\sin^{-1}(\frac{\sqrt{2}}{7}) = \theta$
 จะได้ $\sin(\theta) = \frac{\sqrt{2}}{7}$



$\tan(\sin^{-1}(\frac{\sqrt{2}}{7})) = \frac{\sqrt{2}}{\sqrt{47}} = \frac{\sqrt{2}\sqrt{47}}{47}$

3.8) $\cos(\tan^{-1}(-\frac{\sqrt{7}}{5})) = \frac{5\sqrt{2}}{8}$

ให้ $\tan^{-1}(-\frac{\sqrt{7}}{5}) = \theta$
 จะได้ $\tan(\theta) = -\frac{\sqrt{7}}{5}$

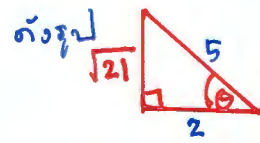


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 จตุภาคที่ 4

$\cos(\tan^{-1}(-\frac{\sqrt{7}}{5})) = \frac{5}{4\sqrt{2}} = \frac{5\sqrt{2}}{8}$

3.9) $\cot(\cos^{-1}(-\frac{2}{5})) = -\frac{2\sqrt{21}}{21}$

ให้ $\cos^{-1}(-\frac{2}{5}) = \theta$
 จะได้ $\cos(\theta) = -\frac{2}{5}$



θ อยู่ใน
 จตุภาคที่ 2

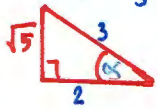
$\cot(\cos^{-1}(-\frac{2}{5})) = -\frac{2}{\sqrt{21}} = -\frac{2\sqrt{21}}{21}$

4. จงหาค่าของ $\sin(\alpha + \beta)$

4.1) $\sin(\cos^{-1}(\frac{2}{3}) + \sin^{-1}(-\frac{5}{9})) =$

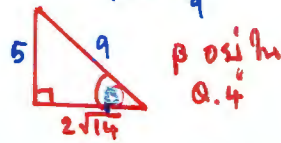
ให้ $\cos^{-1}(\frac{2}{3}) = \alpha$

$\cos(\alpha) = \frac{2}{3}$



ให้ $\sin^{-1}(-\frac{5}{9}) = \beta$

$\sin(\beta) = -\frac{5}{9}$



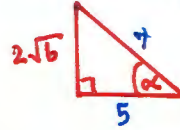
$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$
 $= \frac{\sqrt{5}}{3} \cdot \frac{2\sqrt{14}}{9} + \frac{2}{3} \cdot (-\frac{5}{9}) = \frac{2\sqrt{5}\sqrt{14}}{27} - \frac{10}{27}$

$\cos(\alpha + \beta)$

4.2) $\cos(\cos^{-1}(\frac{5}{7}) + \sin^{-1}(\frac{5}{9})) =$

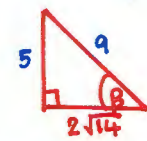
ให้ $\cos^{-1}(\frac{5}{7}) = \alpha$

$\cos(\alpha) = \frac{5}{7}$



ให้ $\sin^{-1}(\frac{5}{9}) = \beta$

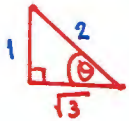
$\sin(\beta) = \frac{5}{9}$



$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$
 $= \frac{5}{7} \cdot \frac{2\sqrt{14}}{9} - \frac{2\sqrt{6}}{7} \cdot \frac{5}{9} = \frac{10\sqrt{14}}{63} - \frac{10\sqrt{6}}{63}$

4.3) $\sin(2\sin^{-1}(\frac{1}{2})) = \frac{\sqrt{3}}{2}$

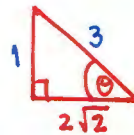
ให้ $\sin^{-1}(\frac{1}{2}) = \theta$ จะได้ $\sin(\theta) = \frac{1}{2}$



ดังนั้น $\sin(2\sin^{-1}(\frac{1}{2})) = \sin(2\theta)$
 $= 2\sin(\theta)\cos(\theta)$
 $= 2(\frac{1}{2})(\frac{\sqrt{3}}{2})$
 $= \frac{\sqrt{3}}{2}$

4.4) $\cos(2\sin^{-1}(\frac{1}{3})) = \frac{7}{9}$

ให้ $\sin^{-1}(\frac{1}{3}) = \theta$ จะได้ $\sin(\theta) = \frac{1}{3}$



ดังนั้น $\cos(2\sin^{-1}(\frac{1}{3})) = \cos(2\theta)$
 $= 2\cos^2(\theta) - 1$
 $= 2(\frac{2\sqrt{2}}{3})^2 - 1$
 $= \frac{7}{9}$

4.5) $\tan^{-1}(\cos(8\pi) + \sin(\frac{23\pi}{2})) = 0$

$\tan^{-1}(\cos(8\pi) + \sin(\frac{23\pi}{2}))$
 $= \tan^{-1}(1 + (-1))$
 $= \tan^{-1}(0)$
 $= 0$

4.6) $\cot(\cos^{-1}(\sin(\frac{13\pi}{6})) + \sin^{-1}(\cos(\frac{8\pi}{3}))) = \sqrt{3}$

$\cot(\cos^{-1}(\sin(\frac{13\pi}{6})) + \sin^{-1}(\cos(\frac{8\pi}{3})))$
 $= \cot(\cos^{-1}(\frac{1}{2}) + \sin^{-1}(-\frac{1}{2}))$
 $= \cot(\frac{\pi}{3} - \frac{\pi}{6})$
 $= \cot(\frac{\pi}{6})$
 $= \sqrt{3}$

5. จงหา Question = $\sin(2\sin^{-1}(\alpha))$

เมื่อ $0 < \alpha \leq 1$

ให้ $\sin^{-1}(\alpha) = A$ จะได้ $\sin(A) = \alpha$



$\sin(2\sin^{-1}(\alpha)) = \sin(2A)$
 $= 2\sin A \cos A$
 $= 2\alpha \sqrt{1-\alpha^2}$

6. จงหา Question = $\cos(2\sin^{-1}(\alpha))$

เมื่อ $0 < \alpha \leq 1$

ให้ $\sin^{-1}(\alpha) = A$ จะได้ $\sin(A) = \alpha$

$\cos(2\sin^{-1}(\alpha)) = \cos(2A)$
 $= 2\cos^2(A) - 1$
 $= 2(\sqrt{1-\alpha^2})^2 - 1$
 $= 2 - 2\alpha^2 - 1$
 $= 1 - 2\alpha^2$

