



แบบฝึกหัดเรื่อง ตรีโกณมิติ

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เลขประจำตัว

1. จงแก้สมการต่อไปนี้ เมื่อ $0 \leq \theta < \frac{\pi}{2}$

1.1) $2 \sin(\theta) - 1 = 0$

$$2 \sin(\theta) - 1 = 0$$

$$\sin(\theta) = \frac{1}{2}$$

$$\theta = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$$

เมื่อ $0 \leq \theta < \frac{\pi}{2}$

จึงได้ $\theta = \frac{\pi}{6}$

1.2) $4 \sin^2(\theta) - 1 = 0$

$$4 \sin^2(\theta) - 1 = 0$$

$$\sin^2(\theta) = \frac{1}{4}$$

$$\sin(\theta) = \pm \frac{1}{2}$$

$$\theta = \frac{\pi}{6} + k\pi, \frac{5\pi}{6} + k\pi$$

เมื่อ $0 \leq \theta < \frac{\pi}{2}$

จึงได้ $\theta = \frac{\pi}{6}$

1.3) $\sin(4\theta) = 0$

$$\sin(4\theta) = 0$$

$$4\theta = 0 + k\pi$$

$$\theta = 0 + \frac{k\pi}{4}$$

เมื่อ $0 \leq \theta < \frac{\pi}{2}$

จึงได้ $\theta = 0, \frac{\pi}{4}$

1.4) $\cos(4\theta) = 1$

$$\cos(4\theta) = 1$$

$$4\theta = 0 + 2k\pi$$

$$\theta = 0 + \frac{k\pi}{2}$$

เมื่อ $0 \leq \theta < \frac{\pi}{2}$

จึงได้ $\theta = 0$

2. จงแก้สมการต่อไปนี้ เมื่อ $0 \leq \theta < 2\pi$

2.1) $2 \cos^2(\theta) - 3 \cos(\theta) + 1 = 0$

$(2 \cos(\theta) - 1)(\cos(\theta) - 1) = 0$

$2 \cos(\theta) - 1 = 0$

$\cos(\theta) = \frac{1}{2}$

$\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

เมื่อ $0 \leq \theta < 2\pi$

จะได้ $\theta = 0, \frac{\pi}{3}, \frac{5\pi}{3}$

$\cos(\theta) - 1 = 0$

$\cos(\theta) = 1$

$\theta = 0 + 2n\pi$

2.2) $2 \cos^2(\theta) + \sin(\theta) - 1 = 0$

$2(1 - \sin^2(\theta)) + \sin(\theta) - 1 = 0$

$2 \sin^2(\theta) - \sin(\theta) - 1 = 0$

$(2 \sin(\theta) + 1)(\sin(\theta) - 1) = 0$

$2 \sin(\theta) + 1 = 0$

$\sin(\theta) = -\frac{1}{2}$

$\theta = \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi$

เมื่อ $0 \leq \theta < 2\pi$

จะได้ $\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\sin(\theta) - 1 = 0$

$\sin(\theta) = 1$

$\theta = \frac{\pi}{2} + 2n\pi$

2.3) $2 \cos^2(\theta) - \cos(\theta) = 0$

$\cos(\theta)[2 \cos(\theta) - 1] = 0$

$\cos(\theta) = 0$

$\theta = \frac{\pi}{2} + n\pi$

เมื่อ $0 \leq \theta < 2\pi$

จะได้ $\theta = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$

$2 \cos(\theta) - 1 = 0$

$\cos(\theta) = \frac{1}{2}$

$\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

2.4) $\sin(\theta) - \cos(\theta) = \sqrt{2}$

$(\sin(\theta) - \cos(\theta))^2 = (\sqrt{2})^2$

$\sin^2(\theta) - 2 \sin(\theta) \cos(\theta) + \cos^2(\theta) = 2$

$1 - 2 \sin(\theta) \cos(\theta) = 2$

$\sin(2\theta) = -1$

$2\theta = \frac{3\pi}{2} + 2n\pi$

$\theta = \frac{3\pi}{4} + n\pi$

เมื่อ $0 \leq \theta < 2\pi$

จะได้ $\theta = \frac{3\pi}{4}$

มุมที่ $\frac{3\pi}{4}$ และ $\frac{7\pi}{4}$ ไม่เป็นจริง

2.5) $\tan(\theta) \sin(\theta) = \sin(\theta)$

$\tan(\theta) \sin(\theta) - \sin(\theta) = 0$

$\sin(\theta) (\tan(\theta) - 1) = 0$

$\sin(\theta) = 0$

$\theta = 0 + n\pi$

เมื่อ $0 \leq \theta < 2\pi$

จะได้ $\theta = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$

$\tan(\theta) = 1$

$\theta = \frac{\pi}{4} + n\pi$

2.6) $4 \cos^3(\theta) - \cos(\theta) = 0$

$\cos(\theta)[4 \cos^2(\theta) - 1] = 0$

$\cos(\theta)(2 \cos(\theta) - 1)(2 \cos(\theta) + 1) = 0$

$\cos(\theta) = 0$

$\theta = \frac{\pi}{2} + n\pi$

$2 \cos(\theta) - 1 = 0$

$\cos(\theta) = \frac{1}{2}$

$\theta = \frac{\pi}{3} + 2n\pi,$

$\frac{5\pi}{3} + 2n\pi$

เมื่อ $0 \leq \theta < 2\pi$

จะได้ $\theta = \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$

$2 \cos(\theta) + 1 = 0$

$\cos(\theta) = -\frac{1}{2}$

$\theta = \frac{2\pi}{3} + 2n\pi,$

$\frac{4\pi}{3} + 2n\pi$

3. จงแก้สมการต่อไปนี้ เมื่อ $0^\circ \leq \theta < 90^\circ$

3.1) $2 \sin(\theta) + 1 = 0$

$$\sin(\theta) = -\frac{1}{2}$$

$$\theta = 210^\circ + n(360^\circ)$$

$$330^\circ + n(360^\circ)$$

เมื่อ $0^\circ \leq \theta < 90^\circ$ ไม่มีมุม θ ที่ทำให้นักสมการเป็นจริง

3.2) $4 \cos^2(\theta) - 3 = 0$

$$\cos^2(\theta) = \frac{3}{4}$$

$$\cos(\theta) = \pm \frac{\sqrt{3}}{2}$$

$$\theta = 30^\circ + n(360^\circ), 150^\circ + n(360^\circ)$$

เมื่อ $0^\circ \leq \theta < 90^\circ$ จะได้ $\theta = 30^\circ$

3.3) $\cos(3\theta) = 0$

$$3\theta = 90^\circ + n(180^\circ)$$

$$\theta = 30^\circ + n(60^\circ)$$

เมื่อ $0^\circ \leq \theta < 90^\circ$

จะได้ $\theta = 30^\circ$

3.4) $\sin(3\theta) = 1$

$$3\theta = 90^\circ + n(360^\circ)$$

$$\theta = 30^\circ + n(120^\circ)$$

เมื่อ $0^\circ \leq \theta < 90^\circ$

จะได้ $\theta = 30^\circ$

4. จงแก้สมการต่อไปนี้ เมื่อ $0^\circ \leq \theta < 360^\circ$

4.1) $2 \sin^2(\theta) + \sin(\theta) - 1 = 0$

$$(2 \sin(\theta) - 1)(\sin(\theta) + 1) = 0$$

$$\sin(\theta) = \frac{1}{2}$$

$$\theta = 30^\circ + n(360^\circ),$$

$$150^\circ + n(360^\circ)$$

$$\sin(\theta) = -1$$

$$\theta = 270^\circ + n(360^\circ)$$

เมื่อ $0^\circ \leq \theta < 360^\circ$ จะได้ $\theta = 30^\circ, 150^\circ, 270^\circ$

4.2) $2 \sin^2(\theta) - 3 \cos(\theta) - 3 = 0$

$$2[1 - \cos^2(\theta)] - 3 \cos(\theta) - 3 = 0$$

$$2 \cos^2(\theta) + 3 \cos(\theta) + 1 = 0$$

$$(2 \cos(\theta) + 1)(\cos(\theta) + 1) = 0$$

$$\cos(\theta) = -\frac{1}{2}$$

$$\theta = 120^\circ + n(360^\circ), 240^\circ + n(360^\circ)$$

$$\cos(\theta) = -1$$

$$\theta = 180^\circ + n(360^\circ)$$

เมื่อ $0^\circ \leq \theta < 360^\circ$ จะได้ $\theta = 120^\circ, 180^\circ, 240^\circ$

4.3) $2 \cos^2(\theta) + \cos(\theta) = 0$

$$\cos(\theta)[2 \cos(\theta) + 1] = 0$$

$$\cos(\theta) = 0$$

$$\theta = 90^\circ + n(180^\circ)$$

$$2 \cos(\theta) + 1 = 0$$

$$\cos(\theta) = -\frac{1}{2}$$

$$\theta = 120^\circ + n(360^\circ),$$

$$240^\circ + n(360^\circ)$$

เมื่อ $0^\circ \leq \theta < 360^\circ$ จะได้ $\theta = 90^\circ, 120^\circ, 240^\circ, 270^\circ$

4.4) $\sin(\theta) + \cos(\theta) = \sqrt{2}$

$$(\sin(\theta) + \cos(\theta))^2 = (\sqrt{2})^2$$

$$\sin^2(\theta) + 2 \sin(\theta) \cos(\theta) + \cos^2(\theta) = 2$$

$$1 + \sin(2\theta) = 2$$

$$\sin(2\theta) = 1$$

$$2\theta = 90^\circ + n(360^\circ)$$

$$\theta = 45^\circ + n(180^\circ)$$

เมื่อ $0^\circ \leq \theta < 360^\circ$ จะได้ $\theta = 45^\circ$ ทำให้นักสมการเป็นจริง

4.5) $\tan(\theta) \sin(\theta) = \tan(\theta)$

$$\tan(\theta) \sin(\theta) - \tan(\theta) = 0$$

$$\tan(\theta) (\sin(\theta) - 1) = 0$$

$$\tan(\theta) = 0$$

$$\theta = 0^\circ + n(180^\circ)$$

$$\sin(\theta) = 1$$

$$\theta = 90^\circ + n(360^\circ)$$

เมื่อ $0^\circ \leq \theta < 360^\circ$ จะได้ $\theta = 0^\circ, 90^\circ, 180^\circ$

4.6) $4 \sin^3(\theta) - 3 \sin(\theta) = 0$

$$\sin(\theta) [4 \sin^2(\theta) - 3] = 0$$

$$\sin(\theta) (2 \sin(\theta) - \sqrt{3})(2 \sin(\theta) + \sqrt{3}) = 0$$

$$\theta = 0^\circ + n(180^\circ), 60^\circ + n(180^\circ), 120^\circ + n(180^\circ)$$

เมื่อ $0^\circ \leq \theta < 360^\circ$

จะได้ $\theta = 0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ$

5. จงแก้สมการต่อไปนี้

5.1) $2 \cos(\theta) - 1 = 0$

$$\cos(\theta) = \frac{1}{2}$$

$$\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$$

5.2) $4 \sin^2(\theta) = 3$

$$\sin^2(\theta) = \frac{3}{4}$$

$$\sin(\theta) = \pm \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$$

5.3) $\tan(\theta) \cos(\theta) = \cos(\theta)$

$$\tan(\theta) \cos(\theta) - \cos(\theta) = 0$$

$$\cos(\theta) (\tan(\theta) - 1) = 0$$

$$\cos(\theta) = 0$$

$$\theta = \frac{\pi}{2} + n\pi$$

$$\tan(\theta) = 1$$

$$\theta = \frac{\pi}{4} + n\pi$$

5.4) $\tan^2(\theta) - 1 = 0$

$$\tan^2(\theta) = 1$$

$$\tan(\theta) = \pm 1$$

$$\theta = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$$

