



แบบฝึกหัดเรื่อง ตรีโกณมิติ

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เลขประจำตัว Trigonometry 06 No. 3.1. จงแก้สมการต่อไปนี้ เมื่อ $0 \leq \theta < \frac{\pi}{2}$

1.1) $2 \sin(\theta) + 1 = 0$

$$2 \sin(\theta) = -1$$

$$\sin(\theta) = -\frac{1}{2}$$

$$\theta = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$\because 0 \leq \theta < \frac{\pi}{2}$$

 \therefore ไม่มีคำตอบ

1.2) $4 \cos^2 \theta - 1 = 0$

$$\cos^2 \theta = \frac{1}{4}$$

$$\cos \theta = \pm \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\pm \frac{1}{2}\right)$$

$$\theta = \frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi$$

$$\because 0 \leq \theta < \frac{\pi}{2}$$

$$\therefore \theta = \frac{\pi}{3}$$

1.3) $\sin(3\theta) = 0$

$$\sin(3\theta) = 0$$

$$3\theta = 0 + 2n\pi, \pi + 2n\pi$$

$$\theta = 0 + \frac{2n\pi}{3}, \frac{\pi}{3} + \frac{2n\pi}{3}$$

$$\because 0 \leq \theta < \frac{\pi}{2}$$

$$\therefore \theta = 0, \frac{\pi}{3}$$

1.4) $\cos(3\theta) = 1$

$$3\theta = 0 + 2n\pi$$

$$\theta = 0 + \frac{2n\pi}{3}$$

$$\because 0 \leq \theta < \frac{\pi}{2}$$

$$\therefore \theta = 0$$

2. จงแก้สมการต่อไปนี้ เมื่อ $0 \leq \theta < 2\pi$

2.1) $2 \sin^2(\theta) - \sin(\theta) - 1 = 0$

ให้ $x = \sin(\theta)$

$2x^2 - x - 1 = 0$

$(2x + 1)(x - 1) = 0$

$x = -\frac{1}{2} \quad x = 1$

$\sin(\theta) = -\frac{1}{2} \quad \sin(\theta) = 1$

$\theta = \frac{7\pi}{6} + 2n\pi, \quad \theta = \frac{\pi}{2} + 2n\pi = \frac{\pi}{2}$

$\frac{11\pi}{6} + 2n\pi$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$

2.3) $2 \sin^2(\theta) - \sin(\theta) = 0$

$\sin(\theta) [2 \sin(\theta) - 1] = 0$

$\sin(\theta) = 0 \quad \sin(\theta) = \frac{1}{2}$
 $\theta = 0 + n\pi \quad \theta = \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi$

$\theta = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

2.5) $2 \tan(\theta) \cos(\theta) = \tan(\theta)$

$\tan(\theta) [2 \cos(\theta) - 1] = 0$

$\tan(\theta) = 0, \quad 2 \cos(\theta) - 1 = 0$
 $\theta = 0 + n\pi \quad \cos(\theta) = \frac{1}{2}$
 $\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

$\theta = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

2.2) $2 \cos^2(\theta) - 3 \sin(\theta) - 3 = 0$

$2 [1 - \sin^2(\theta)] - 3 \sin(\theta) - 3 = 0$

$2 - 2 \sin^2(\theta) - 3 \sin(\theta) - 3 = 0$

$2 \sin^2(\theta) + 3 \sin(\theta) + 1 = 0$

$(2 \sin(\theta) + 1)(\sin(\theta) + 1) = 0$

$\sin(\theta) = -\frac{1}{2} \quad \sin(\theta) = -1$

$\theta = \frac{7\pi}{6} + 2n\pi \quad \theta = \frac{3\pi}{2} + 2n\pi$

$\frac{11\pi}{6} + 2n\pi$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}$

2.4) $\sin(\theta) + \cos(\theta) = \sqrt{2}$

$[\sin(\theta) + \cos(\theta)]^2 = (\sqrt{2})^2$

$\sin^2(\theta) + 2 \sin(\theta) \cos(\theta) + \cos^2(\theta) = 2$

$\sin^2(\theta) + \cos^2(\theta) + 2 \sin(\theta) \cos(\theta) = 2$

$1 + \sin(2\theta) = 2$

$\sin(2\theta) = 1$

$2\theta = \frac{\pi}{2} + 2n\pi$

$\theta = \frac{\pi}{4} + n\pi$

$\theta = \frac{\pi}{4}$

2.6) $4 \sin^3(\theta) - 3 \sin(\theta) = 0$

$\sin(\theta) [4 \sin^2(\theta) - 3] = 0$

$\sin(\theta) = 0 \quad \sin^2(\theta) = \frac{3}{4}$
 $\theta = 0 + n\pi \quad \sin(\theta) = \pm \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

$\frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$

$\theta = 0, \pi, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

3. จงแก้สมการต่อไปนี้ เมื่อ $0^\circ \leq \theta < 90^\circ$

3.1) $2 \sin(\theta) - 1 = 0$

$\sin(\theta) = \frac{1}{2}$
 $\theta = 30^\circ + 360^\circ n$
 $\theta = 30^\circ$

3.2) $4 \cos^2(\theta) - 3 = 0$

$\cos^2 \theta = \frac{3}{4}$
 $\cos \theta = \pm \frac{\sqrt{3}}{2}$
 $\theta = 30^\circ + 360^\circ n, 150^\circ + 360^\circ n, 210^\circ + 360^\circ n, 330^\circ + 360^\circ n$
 $\theta = 30^\circ$

3.3) $\cos(4\theta) = 0$

$4\theta = 90^\circ + 360^\circ n, 270^\circ + 360^\circ n$
 $\theta = 22.5^\circ + 90^\circ n, 67.5^\circ + 90^\circ n$
 $\theta = 22.5^\circ, 67.5^\circ$

3.4) $\sin(2\theta) = 1$

$2\theta = 90^\circ + 360^\circ n$
 $\theta = 45^\circ + 180^\circ n$
 $\theta = 45^\circ$

4. จงแก้สมการต่อไปนี้ เมื่อ $0^\circ \leq \theta < 360^\circ$

4.1) $2 \cos^2(\theta) + \cos(\theta) - 1 = 0$

$(2 \cos(\theta) - 1)(\cos(\theta) + 1) = 0$
 $\cos(\theta) = \frac{1}{2} \quad \cos(\theta) = -1$
 $\theta = 60^\circ + 360^\circ n, 300^\circ + 360^\circ n$
 $\theta = 180^\circ + 360^\circ n$
 $\theta = 60^\circ, 180^\circ, 300^\circ$

4.2) $2 \sin^2(\theta) + 3 \cos(\theta) - 3 = 0$

$2[1 - \cos^2(\theta)] + 3 \cos(\theta) - 3 = 0$
 $2 \cos^2(\theta) - 3 \cos(\theta) + 1 = 0$
 $(2 \cos(\theta) - 1)(\cos(\theta) - 1) = 0$
 $\cos(\theta) = \frac{1}{2} \quad \cos(\theta) = 1$
 $\theta = 60^\circ + 360^\circ n, 300^\circ + 360^\circ n$
 $\theta = 0^\circ + 360^\circ n$
 $\theta = 0^\circ, 60^\circ, 300^\circ$

4.3) $2 \sin^2(\theta) + \sin(\theta) = 0$

$\sin(\theta)(2 \sin(\theta) + 1) = 0$
 $\sin(\theta) = 0 \quad \sin(\theta) = -\frac{1}{2}$
 $\theta = 0^\circ + 360^\circ n, 180^\circ + 360^\circ n$
 $\theta = 210^\circ + 360^\circ n, 330^\circ + 360^\circ n$
 $\theta = 0^\circ, 180^\circ, 210^\circ, 330^\circ$

4.4) $\cos(\theta) - \sin(\theta) = \sqrt{2}$

4.5) $\tan(\theta) \sin(\theta) = \tan(\theta)$

$\frac{\tan(\theta) \sin(\theta)}{\tan(\theta)} = \frac{\tan(\theta)}{\tan(\theta)}$ | $\tan(\theta) = 0$
 $\sin(\theta) = 1$ | $\theta = 0^\circ + 360^\circ n$
 $\theta = 90^\circ + 360^\circ n$ | $\theta = 180^\circ + 360^\circ n$
 $\theta = 0^\circ, 90^\circ, 180^\circ$

4.6) $4 \sin^3(\theta) - \sin(\theta) = 0$

$\sin(\theta)[4 \sin^2(\theta) - 1] = 0$
 $\sin(\theta) = 0$ | $\sin^2(\theta) = \frac{1}{4}$
 $\theta = 0^\circ + 360^\circ n, 180^\circ + 360^\circ n$ | $\sin(\theta) = \pm \frac{1}{2}$
 $\theta = 30^\circ + 360^\circ n, 150^\circ + 360^\circ n, 210^\circ + 360^\circ n, 330^\circ + 360^\circ n$
 $\theta = 0^\circ, 30^\circ, 150^\circ, 180^\circ, 210^\circ, 330^\circ$

5. จงแก้สมการต่อไปนี้

5.1) $2 \cos(\theta) - 1 = 0$

$$\cos(\theta) = \frac{1}{2}$$

$$\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$$

5.2) $4 \sin^2(\theta) = 1$

$$\sin^2(\theta) = \frac{1}{4}$$

$$\sin(\theta) = \pm \frac{1}{2}$$

$$\theta = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$$

5.3) $\tan(\theta) \sin(\theta) = \sin(\theta)$

$$\frac{\tan(\theta) \sin(\theta)}{\sin(\theta)} = \frac{\sin(\theta)}{\sin(\theta)}$$

$$\tan(\theta) = 1$$

$$\theta = \frac{\pi}{4} + 2n\pi, \frac{5\pi}{4} + 2n\pi$$

หรือ $\sin(\theta) = 0$

$$\theta = 0 + 2n\pi, \pi + 2n\pi$$

ดังนั้น $\theta = 0 + 2n\pi, \frac{\pi}{4} + 2n\pi,$

$$\frac{5\pi}{4} + 2n\pi, \pi + 2n\pi$$

5.4) $\tan^2(\theta) - 1 = 0$

$$\tan^2(\theta) = 1$$

$$\tan(\theta) = \pm 1$$

$$\theta = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$$

