แบบฝึกหัดเรื่อง ตรีโกณมิติ
ชื่อ-นามสกุล $\qquad$
เลขประจำตัว $\qquad$ TrigonumetryO6 No. 3.

1. จงแก้สมการต่อไปนี้ เมื่อ $0 \leq \theta<\frac{\pi}{2}$
1.1)

$$
\begin{aligned}
2 \sin (\theta)+1 & =0 \\
2 \sin (\theta) & =-1 \\
\sin (\theta) & =-\frac{1}{2} \\
\theta & =\sin ^{-1}\left(-\frac{1}{2}\right)
\end{aligned}
$$

$$
\therefore \quad 0 \leqslant \theta<\frac{\pi}{2}
$$

$\therefore$ 万ม่มัตต๐พ
1.3)

$$
\begin{aligned}
& \text { 1.3) } \begin{aligned}
\sin (3 \theta) & =0 \\
\sin (3 \theta) & =0 \\
3 \theta & =0+\frac{2 n \pi}{}=\pi+2 n \pi \\
\theta & =0+\frac{2 n \pi}{3}, \frac{\pi}{3}+\frac{2 n \pi}{3}
\end{aligned} .
\end{aligned}
$$

$$
\because \quad 0 \leq \theta<\frac{\pi}{2}
$$

$$
\therefore \quad \theta=0, \frac{\pi}{3}
$$

1.2)


$$
\begin{aligned}
\cos \theta & = \pm \frac{1}{2} \\
\theta & =\cos ^{-1}\left( \pm \frac{1}{2}\right) \\
\theta & =\frac{\pi}{3}+2 n \pi, \frac{2 \pi}{3}+\operatorname{con} \pi \\
\because \quad 0 & \leqslant \theta<\frac{\pi}{2} \\
\therefore \quad \theta & =\frac{\pi}{3}
\end{aligned}
$$

1.4)

$$
\begin{aligned}
\cos (3 \theta) & =1 \\
3 \theta & =0+2 n \pi \\
\theta & =0+\frac{2 n \pi}{3} \\
\therefore \quad 0 \leqslant \theta & <\frac{\pi}{2} \\
\therefore \quad \theta & =0
\end{aligned}
$$

2. จงแก้สมการต่อไปนี้ เมื่อ $0 \leq \theta<2 \pi$
2.1) $2 \sin ^{2}(\theta)-\sin (\theta)-1=0$

$$
\begin{aligned}
& 9 x x=\sin (\theta) \\
& 2 x^{2}-x-1=0 \\
& (2 x+1)(x-1)=0 \\
& x=-\frac{1}{2} \quad x=1 \\
& \sin (\theta)=-\frac{1}{2} \quad \sin (\theta)=1 \\
& \theta=\frac{7 \pi}{6}+2 n \pi, \quad \theta=\frac{\pi}{2}+2 n \pi=\frac{\pi}{2}+2 n \pi \quad \theta=\frac{7 \pi}{6}, \frac{11 \pi}{6}, \frac{\pi}{2}
\end{aligned}
$$

$$
\text { 2.3) } 2 \sin ^{2}(\theta)-\sin (\theta)=0
$$

$$
\sin (\theta)[2 \sin (\theta)-1]=0
$$

$$
\begin{array}{rlrl}
\sin (\theta) & =0 & \sin (\theta) & =\frac{1}{2} \\
\theta & =0+n \pi & \theta & =\frac{\pi}{6}+2 n \pi \\
& \frac{5 \pi}{6}+2 n \pi
\end{array}
$$

$$
\theta=0, \pi, \frac{\pi}{6}, \frac{5 \pi}{6}
$$

2.5) $2 \tan (\theta) \cos (\theta)=\tan (\theta)$

$$
\tan (\theta)[2 \cos (\theta)-1]=0
$$

$$
\begin{array}{rlrl}
\tan (\theta)=0, & 2 \cos (\theta)-1 & =0 \\
\theta=0+n \pi, & \cos (\theta) & =\frac{1}{2} \\
\theta & =\frac{\pi}{3}+2 n \pi \\
& & \frac{5 \pi}{3}+2 n \pi
\end{array}
$$

$$
\theta=0, \pi, \frac{\pi}{3}, \frac{5 \pi}{3}
$$

2.2) $\square$

$$
2 \cos ^{2}(\theta)-3 \sin (\theta)-3=0
$$

$$
\begin{aligned}
& 2\left[1-\sin ^{2}(\theta)\right]-3 \sin (\theta)-3=0 . \\
& 2-2 \sin ^{2}(\theta)-3 \sin (\theta)-3=0 . \\
& 2 \sin ^{2}(\theta)+3 \sin (\theta)+1=0 . \\
& (2 \sin (\theta)+1)(\sin (\theta)+1)=0 .
\end{aligned}
$$

$$
\sin (\theta)=-\frac{1}{2} \quad \sin (\theta)=-1
$$

$$
\begin{array}{ll}
\theta=\frac{1 \pi}{6}+2 n \pi & \theta=\frac{3 \pi}{2}+2 n \pi \\
\frac{11 \pi}{6}+2 n \pi & \theta
\end{array}
$$

2.4)

$$
\begin{aligned}
& {[\sin (\theta)+\cos (\theta)=\sqrt{2}} \\
& {[\sin (\theta)+\cos (\theta)]^{2}=(\sqrt{2})^{2}} \\
& \sin ^{2}(\theta)+2 \sin (\theta) \cos (\theta)+\cos ^{2}(\theta)=2 \\
& \frac{\sin ^{2}(\theta)+\cos ^{2}(\theta)+2 \sin (\theta) \cos (\theta)=2}{\downarrow}=2 \\
& 1+\sin ^{2}(2 \theta)=2 \\
& \sin (2 \theta)=1 \\
& 2 \theta=\frac{\pi}{2}+2 n \pi \\
& \theta=\frac{\pi}{4}+n \pi
\end{aligned}
$$

2.6) $4 \sin ^{3}(\theta)-3 \sin (\theta)=0$.

$$
\begin{aligned}
& \sin (\theta)\left[4 \sin ^{2}(\theta)-3\right]=0 \\
& \sin (\theta)=0 \\
& \theta=0+n \pi \left\lvert\, \sin (\theta)=\frac{3}{4}\right. \\
& \theta=\frac{\sqrt{3}}{2} \\
& =\frac{\pi}{3}+2 n \pi, \frac{5 \pi}{3}+2 n \pi \\
& \\
& \frac{2 \pi}{3}+2 n \pi, \frac{4 \pi}{3}+2 n \pi
\end{aligned}
$$

$$
\theta=0, \pi, \frac{\pi}{3}, \frac{2 \pi}{3}, \frac{4 \pi}{3}, \frac{5 \pi}{3}
$$

3. จงแก้สมการต่อไปนี้ เมื่อ $0^{\circ} \leq \theta<90^{\circ}$

$\sin (\theta)=\frac{1}{2}$
$\theta=30^{\circ}+360^{\circ} n$
$\theta=30^{\circ}$
3.3) $\cos (4 \theta)=0$
$4 \theta=90^{\circ}+360^{\circ} n, 270^{\circ}+360^{\circ} n$
$\theta=22.5^{\circ}+90^{\circ} n, 67.5^{\circ}+90^{\circ} n$
$\theta=22.5^{\circ}, 67.5^{\circ}$
3.2)

$$
\begin{aligned}
4 \cos ^{2}(\theta)-3 & =0 \\
\cos ^{2} \theta & =\frac{3}{4} \\
\cos \theta & = \pm \frac{\sqrt{3}}{2} \\
\theta & =30^{\circ}+360^{\circ} \mathrm{n}, 150^{\circ}+360^{\circ} \mathrm{n}
\end{aligned}
$$

$$
\theta=30^{\circ} \quad 210^{\circ}+360^{\circ} n, 330^{\circ}+360^{\circ} n
$$

3.4)

$$
\begin{aligned}
& \sin (2 \theta)=1 \\
& 2 \theta=90^{\circ}+360^{\circ} n \\
& \theta=45^{\circ}+180^{\circ} n \\
& \theta=45^{\circ}
\end{aligned}
$$

4. จงแก้สมการต่อไปนี้ เมื่อ $0^{\circ} \leq \theta<360^{\circ}$
4.1)

$$
2 \cos ^{2}(\theta)+\cos (\theta)-1=0
$$

$(2 \cos (\theta)-1)(\cos (\theta)+1)=0$
$\cos (\theta)=\frac{1}{2}$
$\cos (\theta)=-1$
$\theta=60^{\circ}+360^{\circ} n$ $300+360 \mathrm{n}$
$\theta=60^{\circ}, 180^{\circ}, 300^{\circ}$
4.3) $2 \sin ^{2}(\theta)+\sin (\theta)=0$
$\sin (\theta)(2 \sin (\theta)+1)=0$
$\sin (\theta)=0$
$\sin (\theta)=-\frac{1}{2}$
$\theta=0^{\circ}+360^{\circ} n$

$$
\theta=210^{\circ}+360^{\circ} n
$$

$$
=180^{\circ}+360^{\circ} n=300^{\circ}+360^{\circ} n
$$

$\theta=0^{\circ}, 180^{\circ}, 210^{\circ} 300^{\circ}$
4.5) $\tan (\theta) \sin (\theta)=\tan (\theta)$
$\begin{aligned} & \tan (\theta) \sin (\theta) \\ & \tan (\theta)=\frac{\tan (-)}{\tan (\theta)}-\left\lvert\, \begin{aligned} \tan (\theta) & =0 \\ \sin (\theta) & =1 \quad\end{aligned} \quad 0^{\circ}+360 n\right. \\ &=180^{\circ}+360^{\circ}\end{aligned}$
$\theta=90^{\circ}+360^{\circ} n$
$\theta=0^{\circ}, 90^{\circ}, 180^{\circ}$
4.2) $2 \sin ^{2}(\theta)+3 \cos (\theta)-3=0$
$?\left[1-\cos ^{2}(\theta)\right]+3 \cos (\theta)-3=0$. $2 \cos ^{2}(\theta)-3 \cos (\theta)+1=0$
$(2 \cos (\theta)-1)(\cos (\theta)-1)=0$
$\cos (\theta)=\frac{1}{2} \cos (\theta)=1$ $\theta=60^{\circ}+360^{\circ} n \quad \theta=0+360^{\circ} n$ $304+360^{\circ} n \quad \theta=0^{\circ} 60^{\circ}, 300^{\circ}$
4.4)

$$
\cos (\theta)-\sin (\theta)=\sqrt{2} .
$$

4.6)

$$
4 \sin ^{3}(\theta)-\sin (\theta)=0
$$

$$
\sin (\theta)\left[4 \sin ^{2}(\theta)-1\right]=0
$$

$$
\sin (\theta)=0 \quad \square \sin ^{2}(\theta)=\frac{1}{4}
$$

$$
\begin{aligned}
\theta & =0^{\circ}+360^{\circ} n \\
& =180^{\circ}+360^{\circ} n
\end{aligned} \left\lvert\, \begin{aligned}
\sin (\theta) & = \pm \frac{1}{2} \\
\theta & =30^{\circ}+360^{\circ} n \\
& =150^{\circ}+360^{\circ} n
\end{aligned}\right.
$$

$$
\theta=0^{\circ}, 30^{\circ}, 150^{\circ}, 180^{\circ}, 210^{\circ}, 330^{\circ}=150^{\circ}+360^{\circ} \mathrm{n} ~=210^{\circ}+360^{\circ} \mathrm{n},
$$

5. จงแก้สมการต่อไปนี้
5.1)

$$
\begin{aligned}
\hline 2 \cos (\theta)-1 & =0 \\
\cos (\theta) & =\frac{1}{2} \\
\theta & =\frac{\pi}{3}+2 n \pi, \frac{5 \pi}{3}+2 n \pi
\end{aligned}
$$

5.3) $\tan (\theta) \sin (\theta)=\sin (\theta)$

$$
\frac{\tan (\theta) \sin (\theta)}{\sin (\theta)}=\frac{\sin (\theta)}{\sin (\theta)}
$$

$$
\tan (\theta)=1
$$

$$
\theta=\frac{\pi}{4}+2 n \pi, \frac{5 \pi}{4}+2 n \pi
$$

$x \geqslant 0 \quad \sin (\theta)=0$

$$
\theta=0+2 n \pi, \pi+2 n \pi
$$

ब二厶मे० $\theta=0+2 n \pi, \frac{\pi}{4}+2 n \pi$,

$$
=\frac{5 \pi}{4}+2 n \pi, \pi+2 n \pi
$$

5.2) $4 \sin ^{2}(\theta)=1$
$\sin ^{2}(\theta)=\frac{1}{4}$
$\sin (\theta)= \pm \frac{1}{2}$
$\theta=\frac{\pi}{6}+n \pi, \frac{5 \pi}{6}+n \pi$
5.4)

$$
\begin{aligned}
& \tan ^{2}(\theta)-1=0 \\
& \tan ^{2}(\theta)=1 \\
& \tan (\theta)= \pm 1
\end{aligned}
$$

$$
\theta=\frac{\pi}{4}+n \pi, \frac{3 \pi}{4}+n \pi
$$

$$
\begin{gathered}
N o l=\left[\begin{array}{cc}
.1=(2 \sin (\theta)+1=0) & .2=\left(4[\cos (\theta)]^{2}-1=0\right) \\
.3=(\sin (3 \theta)=0) & .4=(\cos (3 \theta)=1)
\end{array}\right] \\
N o 2=\left[\begin{array}{cc}
.1=\left(2[\sin (\theta)]^{2}-\sin (\theta)-1=0\right) & .2=\left(2[\cos (\theta)]^{2}-3 \sin (\theta)-3=0\right) \\
.3=\left(2[\sin (\theta)]^{2}-\sin (\theta)=0\right) & .4=(\sin (\theta)+\cos (\theta)=\sqrt{2}) \\
.5=(2 \tan (\theta) \cos (\theta)=\tan (\theta)) & .6=\left(4[\sin (\theta)]^{3}-3 \sin (\theta)=0\right)
\end{array}\right] \\
N o 3=\left[\begin{array}{cc}
.1=(2 \sin (\theta)-1=0) & .2=\left(4[\cos (\theta)]^{2}-3=0\right) \\
.3=(\cos (4 \theta)=0) & .4=(\sin (2 \theta)=1)
\end{array}\right] \\
N o 4=\left[\begin{array}{cc}
.1=\left(2[\cos (\theta)]^{2}+\cos (\theta)-1=0\right) & .2=\left(2[\sin (\theta)]^{2}+3 \cos (\theta)-3=0\right) \\
.3=\left(2[\sin (\theta)]^{2}+\sin (\theta)=0\right) & .4=(\cos (\theta)-\sin (\theta)=\sqrt{2}) \\
.5=(\tan (\theta) \sin (\theta)=\tan (\theta)) & .6=\left(4[\sin (\theta)]^{3}-\sin (\theta)=0\right)
\end{array}\right] \\
N o 5=\left[\begin{array}{cc}
.1=(2 \cos (\theta)-1=0) & .2=\left(4[\sin (\theta)]^{2}=1\right) \\
.3=(\tan (\theta) \sin (\theta)=\sin (\theta)) & .4=\left([\tan (\theta)]^{2}-1=0\right)
\end{array}\right]
\end{gathered}
$$

X Math@MUT XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX6300106-00003XX TrigonometryExercise6 Answers for No. 3

$$
\begin{aligned}
& \text { Ansl }=\left[\begin{array}{cc}
. l=[~] & .2=\left[\frac{\pi}{3}\right] \\
.3=\left[0, \frac{\pi}{3}\right] & .4=[0]
\end{array}\right], \quad,\left[\begin{array}{cc}
M & @ \\
a & M \\
t & U \\
h & T \\
:) & :)
\end{array}\right] \\
& \text { Ans } 2=\left[\begin{array}{lc}
.1=\left[\frac{\pi}{2}, \frac{7 \pi}{6}, \frac{11 \pi}{6}\right] & .2=\left[\frac{7 \pi}{6}, \frac{3 \pi}{2}, \frac{11 \pi}{6}\right] \\
.3=\left[0, \frac{\pi}{6}, \frac{5 \pi}{6}, \pi\right] & .4=\left[\frac{\pi}{4}\right] \\
.5=\left[0, \frac{\pi}{3}, \pi, \frac{5 \pi}{3}\right] & .6=\left[0, \frac{\pi}{3}, \frac{2 \pi}{3}, \pi, \frac{4 \pi}{3}, \frac{5 \pi}{3}\right]
\end{array}\right] \\
& \text { Ans } 3=\left[\begin{array}{cc}
.1=\left[30^{\circ}\right] & .2=\left[30^{\circ}\right] \\
.3=\left[22.5^{\circ}, 67.5^{\circ}\right] & .4=\left[45^{\circ}\right]
\end{array}\right], \quad,\left[\begin{array}{cc}
M & @ \\
a & M \\
t & U \\
h & T
\end{array}\right] \\
& \text { Ans } 4=\left[\begin{array}{cc}
.1=\left[60^{\circ}, 180^{\circ}, 300^{\circ}\right] & .2=\left[0,60^{\circ}, 300^{\circ}\right] \\
.3=\left[0,180^{\circ}, 210^{\circ}, 330^{\circ}\right] & .4=\left[315^{\circ}\right] \\
.5=\left[0,90^{\circ}, 180^{\circ}\right] & .6=\left[0,30^{\circ}, 150^{\circ}, 180^{\circ}, 210^{\circ}, 330^{\circ}\right]
\end{array}\right] \\
& A n s 5=\left[\begin{array}{c}
.1=\left[\frac{1}{3} \pi+2 n \pi, \frac{5}{3} \pi+2 n \pi\right] \\
.3=\left[2 n \pi, \frac{1}{4} \pi+2 n \pi, \pi+2 n \pi, \frac{5}{4} \pi+2 n \pi\right] .4=\left[\frac{1}{6} \pi+n \pi, \frac{5}{6} \pi+n \pi\right] \\
\left..4 \pi, \frac{3}{4} \pi+n \pi\right]
\end{array}\right]
\end{aligned}
$$

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