แบบฝึกหัดเรื่อง อนุพันธ์

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เลขประจำตัว No. 1

1. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ โดยใช้นิยามของอนุพันธ์ $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

1.1) $f(x) = \boxed{x^2+5x-2}$

$$f'(x) = \lim_{h\to 0}\frac{[(x+h)^2+5(x+h)-2]-[x^2+5x-2]}{h}$$

$$= \lim_{h\to 0}\frac{(\cancel{x^2}+2xh+h^2+\cancel{5x}+5h-\cancel{2}-\cancel{x^2}-\cancel{5x}+\cancel{2})}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}(2x+h+5)}{\cancel{h}}$$

$$= 2x+5$$

1.2) $f(x) = \boxed{4x^3+5}$

$$f'(x) = \lim_{h\to 0}\frac{[4(x+h)^3+5]-[4x^3+5]}{h}$$

$$= \lim_{h\to 0}\frac{[4(x^3+3x^2h+3xh^2+h^3)+5]-[4x^3+5]}{h}$$

$$= \lim_{h\to 0}\frac{[\cancel{4x^3}+12x^2h+12xh^2+4h^3+\cancel{5}-\cancel{4x^3}-\cancel{5}]}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}(12x^2+12xh+4h^2)}{\cancel{h}} = 12x^2$$

1.3) $f(x) = \boxed{\frac{3}{x}}$

$$f'(x) = \lim_{h\to 0}\frac{\frac{3}{x+h}-\frac{3}{x}}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{3x}-\cancel{3x}-3h}{h(x)(x+h)}$$

$$= \lim_{h\to 0}\frac{-3\cancel{h}}{\cancel{h}x(x+h)} = -\frac{3}{x^2}$$

1.4) $f(x) = \boxed{2x^{\frac{1}{3}}}$

$$f'(x) = \lim_{h\to 0}\frac{2\sqrt[3]{x+h}-2\sqrt[3]{x}}{h}\cdot\frac{(\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2})}{(\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2})}$$

$$= \lim_{h\to 0}\frac{2}{h}\cdot\frac{(\sqrt[3]{(x+h)^3}-\sqrt[3]{x^3})}{\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2}}$$

$$= \lim_{h\to 0}\frac{2}{\cancel{h}}\cdot\frac{(\cancel{x}+\cancel{h}-\cancel{x})}{\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2}}$$

$$= \frac{2}{\sqrt[3]{x^2}+\sqrt[3]{x^2}+\sqrt[3]{x^2}} = \frac{2}{3x^{\frac{2}{3}}}$$

2. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ ณ จุดที่กำหนดให้ โดยใช้นิยามของอนุพันธ์ $f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

2.1) $f(x) = \boxed{4x^3-5}$, $a = \boxed{5}$

$$f'(a) = \lim_{x\to 5}\frac{(4x^3-5)-(4(5^3)-5)}{x-5}$$

$$= \lim_{x\to 5}\frac{4x^3-500}{x-5}$$

$$= \lim_{x\to 5}\frac{4(x^3-5^3)}{x-5}$$

$$= \lim_{x\to 5}\frac{4\cancel{(x-5)}(x^2+5x+25)}{\cancel{x-5}}$$

$$= 300$$

2.2) $f(x) = \boxed{\frac{6}{x^2}}$, $a = \boxed{-1}$

$$f'(a) = \lim_{x\to -1}\frac{\frac{6}{x^2}-\frac{6}{(-1)^2}}{x+1}$$

$$= \lim_{x\to -1}\frac{\frac{6}{x^2}-6}{x+1}$$

$$= \lim_{x\to -1}\frac{6-6x^2}{x^2(x+1)}$$

$$= \lim_{x\to -1}\frac{6(1-x)\cancel{(1+x)}}{x^2\cancel{(x+1)}}$$

$$= 12$$

3. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ โดยใช้สูตร

3.1) $f(x) = \boxed{x^2+5x-2}$

$f'(x) = 2x+5$

3.2) $f(x) = \boxed{4x^3+5}$

$f'(x) = 12x^2$

3.3) $f(x) = \boxed{2x^5+x^2+3x+5}$

$f'(x) = 10x^4+2x+3$

3.4) $f(x) = \boxed{\frac{4}{3}x^5+\frac{4}{5}x^4+\frac{1}{3}}$

$f'(x) = \frac{20}{3}x^4+\frac{16}{5}x^3$

3.5) $f(x) = \boxed{\frac{2}{\sqrt{x}}+4\sqrt{x}+5x}$

$f'(x) = \frac{d}{dx}\left(\frac{2}{x^{\frac{1}{2}}}+4x^{\frac{1}{2}}+5x\right)$

$= -\frac{1}{x^{\frac{3}{2}}}+\frac{2}{\sqrt{x}}+5$

3.6) $f(x) = \boxed{x^{\frac{3}{4}}+\frac{1}{x^{\frac{3}{4}}}+x^{\frac{1}{4}}+\frac{1}{x^{\frac{1}{4}}}}$

$f'(x) = \frac{d}{dx}\left(x^{\frac{3}{4}}+x^{-\frac{3}{4}}+x^{\frac{1}{4}}+x^{-\frac{1}{4}}\right)$

$= \frac{3}{4}x^{-\frac{1}{4}}-\frac{3}{4}x^{-\frac{7}{4}}+\frac{1}{4}x^{-\frac{3}{4}}-\frac{1}{4}x^{-\frac{5}{4}}$

$= \frac{3}{4x^{\frac{1}{4}}}-\frac{3}{4x^{\frac{7}{4}}}+\frac{1}{4x^{\frac{3}{4}}}-\frac{1}{4x^{\frac{5}{4}}}$

3.7) $f(x) = \boxed{(2x+2)(3x^2+5x+2)}$

$f'(x) = \frac{d}{dx}((2x+2)(3x^2+5x+2))$

$= \frac{d}{dx}(6x^3+16x^2+14x+4)$

$= 18x^2+32x+14$

3.8) $f(x) = \boxed{(x+4)(\sqrt{x}+3)}$

$f'(x) = \frac{d}{dx}((x+4)(\sqrt{x}+3))$

$= \frac{d}{dx}(x^{\frac{3}{2}}+3x+4x^{\frac{1}{2}}+12)$

$= \frac{3}{2}x^{\frac{1}{2}}+3+2x^{-\frac{1}{2}}$

$= \frac{3\sqrt{x}}{2}+\frac{2}{\sqrt{x}}+3$

3.9) $f(x) = \boxed{\dfrac{4x+3}{x-3}}$

$$f'(x) = \frac{d}{dx}\left(\frac{4x+3}{x-3}\right)$$

$$= \frac{(x-3)\frac{d}{dx}(4x+3) - (4x+3)\frac{d}{dx}(x-3)}{(x-3)^2}$$

$$= \frac{(x-3)(4) - (4x+3)(1)}{(x-3)^2}$$

$$= \frac{4x-12-4x-3}{(x-3)^2}$$

$$= -\frac{15}{(x-3)^2}$$

3.10) $f(x) = \boxed{\dfrac{3x^7 - 2x^3}{\sqrt{x}}}$

$$f'(x) = \frac{d}{dx}\left(\frac{3x^7-2x^3}{\sqrt{x}}\right)$$

$$= \frac{d}{dx}\left(3x^{\frac{13}{2}} - 2x^{\frac{5}{2}}\right)$$

$$= \frac{39}{2}x^{\frac{11}{2}} - 5x^{\frac{3}{2}}$$

3.11) $f(x) = \boxed{\left(\frac{1}{x^5} + \frac{2}{x^2}\right)(4x^4+5)}$

$$f'(x) = \frac{d}{dx}\left(\left(\frac{1}{x^5} + \frac{2}{x^2}\right)(4x^4+5)\right)$$

$$= \frac{d}{dx}\left(4x^{-1} + 5x^{-5} + 8x^2 + 10x^{-2}\right)$$

$$= -4x^{-2} - 25x^{-6} + 16x - 20x^{-3}$$

$$= -\frac{4}{x^2} - \frac{25}{x^6} + 16x - \frac{20}{x^3}$$

3.12) $f(x) = \boxed{(4x^5+3x^4)\left(\frac{x+2}{x-3}\right)}$

$$f'(x) = \frac{d}{dx}\left(\frac{4x^6+11x^5+6x^4}{(x-3)}\right)$$

$$= \frac{(x-3)\frac{d}{dx}(4x^6+11x^5+6x^4) - (4x^6+11x^5+6x^4)\frac{d}{dx}(x-3)}{(x-3)^2}$$

$$= \frac{(x-3)(24x^5+55x^4+24x^3) - (4x^6+11x^5+6x^4)(1)}{(x-3)^2}$$

$$= \frac{24x^6+55x^5+24x^4-72x^5-165x^4-72x^3-4x^6-11x^5-6x^4}{(x-3)^2}$$

$$= \frac{20x^6-28x^5-147x^4-72x^3}{(x-3)^2}$$

4. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ ณ จุดที่กำหนดให้

4.1) $f(x) = \boxed{2x^5+x^2+3x+5}$, $a = \boxed{2}$

$$f'(x) = 10x^4 + 2x + 3$$

$$f'(a) = 10(2^4) + 2(2) + 3$$

$$= 167$$

4.2) $f(x) = \boxed{\frac{2}{\sqrt{x}} + 4\sqrt{x} + 5x}$, $a = \boxed{4}$

$$f'(x) = \frac{d}{dx}\left(2x^{-\frac{1}{2}} + 4x^{\frac{1}{2}} + 5x\right)$$

$$= -x^{-\frac{3}{2}} + 2x^{-\frac{1}{2}} + 5$$

$$= -\frac{1}{\sqrt{x^3}} + \frac{2}{\sqrt{x}} + 5$$

$$f'(a) = -\frac{1}{\sqrt{4^3}} + \frac{2}{\sqrt{4}} + 5$$

$$= -\frac{1}{8} + 1 + 5$$

$$= \frac{47}{8}$$

Diff02 for No.1

$$No01 = \begin{bmatrix} .1 = (\mathrm{f}(x) = x^2 + 5\,x - 2) \\ .2 = (\mathrm{f}(x) = 4\,x^3 + 5) \\ .3 = \left(\mathrm{f}(x) = \frac{3}{x}\right) \\ .4 = (\mathrm{f}(x) = 2\,x^{(1/3)}) \end{bmatrix}, \quad , No02 = \begin{bmatrix} .1 = [\mathrm{f}(x) = 4\,x^3 - 5, a = 5] \\ .2 = \left[\mathrm{f}(x) = \frac{6}{x^2}, a = -1\right] \end{bmatrix}$$

$$No03 = \begin{bmatrix} .1 = (\mathrm{f}(x) = x^2 + 5\,x - 2) & .2 = (\mathrm{f}(x) = 4\,x^3 + 5) \\ .3 = (\mathrm{f}(x) = 2\,x^5 + x^2 + 3\,x + 5) & .4 = \left(\mathrm{f}(x) = \frac{4}{3}x^5 + \frac{4}{5}x^4 + \frac{1}{3}\right) \\ .5 = \left(\mathrm{f}(x) = \frac{2}{\sqrt{x}} + 4\sqrt{x} + 5\,x\right) & .6 = \left(\mathrm{f}(x) = x^{(3/4)} + \frac{1}{x^{(3/4)}} + x^{(1/4)} + \frac{1}{x^{(1/4)}}\right) \\ .7 = (\mathrm{f}(x) = (2\,x + 2)\,(3\,x^2 + 5\,x + 2)) & .8 = (\mathrm{f}(x) = (x + 4)\,(\sqrt{x} + 3)) \\ .9 = \left(\mathrm{f}(x) = \frac{3 + 4\,x}{-3 + x}\right) & .10 = \left(\mathrm{f}(x) = \frac{3\,x^7 - 2\,x^3}{\sqrt{x}}\right) \\ .11 = \left(\mathrm{f}(x) = \left(\frac{1}{x^5} + \frac{2}{x^2}\right)(4\,x^4 + 5)\right) & .12 = \left(\mathrm{f}(x) = (4\,x^5 + 3\,x^4)\left[\frac{x + 2}{-3 + x}\right]\right) \end{bmatrix}, \quad , \begin{bmatrix} \begin{bmatrix}\begin{bmatrix} P \\ V \\ S \\ S \end{bmatrix}\end{bmatrix} \\ \& \\ \begin{bmatrix}\begin{bmatrix} M \\ a \\ t \\ h \\ @ \\ M \\ U \\ T \end{bmatrix}\end{bmatrix} \end{bmatrix}$$

$$No04 = \begin{bmatrix} .1 = [\mathrm{f}(x) = 2\,x^5 + x^2 + 3\,x + 5, a = 2] \\ .2 = \left[\mathrm{f}(x) = \frac{2}{\sqrt{x}} + 4\sqrt{x} + 5\,x, a = 4\right] \end{bmatrix}$$

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Diff02 Answers for No.1

$$Ans1 = \begin{bmatrix} .1 = (f'(x) = 2x+5) \\ .2 = (f'(x) = 12x^2) \\ .3 = \left(f'(x) = -\frac{3}{x^2}\right) \\ .4 = \left(f'(x) = \frac{2}{3x^{(2/3)}}\right) \end{bmatrix}, \quad , \quad Ans2 = \begin{bmatrix} .1 = (f'(5) = 300) \\ .2 = (f'(-1) = 12) \end{bmatrix}, \quad , \quad Ans4 = \begin{bmatrix} .1 = [f'(2) = 167] \\ .2 = \left[f'(4) = \frac{47}{8}\right] \end{bmatrix}$$

$$Ans3 = \begin{bmatrix} .1 = (f'(x) = 2x+5) & .2 = (f'(x) = 12x^2) \\ .3 = (f'(x) = 10x^4 + 2x + 3) & .4 = \left(f'(x) = \frac{20}{3}x^4 + \frac{16}{5}x^3\right) \\ .5 = \left(f'(x) = -\frac{1}{x^{(3/2)}} + \frac{2}{\sqrt{x}} + 5\right) & .6 = \left(f'(x) = \frac{3}{4x^{(1/4)}} - \frac{3}{4x^{(7/4)}} + \frac{1}{4x^{(3/4)}} - \frac{1}{4x^{(5/4)}}\right) \\ .7 = (f'(x) = 18x^2 + 32x + 14) & .8 = \left(f'(x) = \frac{3x + 6\sqrt{x} + 4}{2\sqrt{x}}\right) \\ .9 = \left(f'(x) = -\frac{15}{(-3+x)^2}\right) & .10 = \left(f'(x) = \frac{x^{(3/2)}(39x^4 - 10)}{2}\right) \\ .11 = \left(f'(x) = \frac{16x^7 - 4x^4 - 20x^3 - 25}{x^6}\right) & .12 = \left(f'(x) = \frac{20x^6 - 28x^5 - 147x^4 - 72x^3}{(-3+x)^2}\right) \end{bmatrix}, \quad , \quad \begin{bmatrix} \underline{:)} \\ :(\\ \left[\begin{bmatrix} P \\ V \\ S \\ S \end{bmatrix}\right] \\ \& \\ \left[\begin{bmatrix} M \\ a \\ t \\ h \\ @ \\ M \\ U \\ T \end{bmatrix}\right] \\ \underline{:)} \\ :(\end{bmatrix}$$

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