แบบฝึกหัดเรื่อง อนุพันธ์

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เลขประจำตัว No.2

1. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ โดยใช้นิยามของอนุพันธ์ $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

1.1) $f(x) = \boxed{x^2+5x-1}$

$$f'(x) = \lim_{h\to 0}\frac{(x+h)^2+5(x+h)-1-(x^2+5x-1)}{h}$$

$$= \lim_{h\to 0}\frac{2xh+h^2+5h}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}(2x+h+5)}{\cancel{h}}$$

$$f'(x) \overset{h\to 0}{=} 2x+5$$

1.2) $f(x) = \boxed{4x^3-5}$

$$f'(x) = \lim_{h\to 0}\frac{4(x+h)^3-5-(4x^3-5)}{h}$$

$$= \lim_{h\to 0}\frac{12x^2h+12xh^2+4h^3}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}(12x^2+12xh+4h^2)}{\cancel{h}}$$

$$f'(x) = 12x^2$$

1.3) $f(x) = \boxed{\frac{5}{x}}$

$$f'(x) = \lim_{h\to 0}\frac{\frac{5}{(x+h)}-\frac{5}{x}}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{5x-5(x+h)}{x(x+h)}\right)$$

$$= \lim_{h\to 0}\frac{1}{\cancel{h}}\left(\frac{-5\cancel{h}}{x^2+xh}\right)$$

$$f'(x) = \frac{-5}{x^2} = -5x^{-2}$$

1.4) $f(x) = \boxed{4x^{1/3}}$

$$f'(x) = \lim_{h\to 0}\frac{4(x+h)^{1/3}-4x^{1/3}}{h}$$

$$= \lim_{h\to 0}\frac{4}{h}\left[\left((x+h)^{1/3}-x^{1/3}\right)\cdot\frac{\left((x+h)^{2/3}+x^{1/3}(x+h)^{1/3}+x^{2/3}\right)}{\left((x+h)^{2/3}+x^{1/3}(x+h)^{1/3}+x^{2/3}\right)}\right]$$

$$= \lim_{h\to 0}\frac{4}{(x+h)^{2/3}+(x+h)^{1/3}\cdot x^{1/3}+x^{2/3}}$$

$$\therefore f'(x) = \frac{4}{3x^{2/3}}$$

2. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ ณ จุดที่กำหนดให้ โดยใช้นิยามของอนุพันธ์ $f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

2.1) $f(x) = \boxed{2x^3+5}$, $a = \boxed{5}$

$$f'(a) = \lim_{h\to 0}\frac{2(x+h)^3+5-(2x^3+5)}{h}$$

$$= \lim_{h\to 0}\frac{6x^2h+6xh^2+2h^3}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}(6x^2+6xh+2h^2)}{\cancel{h}}$$

$$f'(x) = 6x^2$$

$$f'(a) = f'(5) = 6(5)^2 = \boxed{150}$$

2.2) $f(x) = \boxed{\frac{5}{x^2}}$, $a = \boxed{-1}$

$$f'(a) = \lim_{h\to 0}\frac{\frac{5}{(x+h)^2}-\frac{5}{x^2}}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{5x^2-5(x+h)^2}{x^2(x+h)^2}\right) = \lim_{h\to 0}\frac{(-10xh-5h^2)}{h(x^2(x+h)^2)}$$

$$f'(x) = \lim_{h\to 0}\frac{\cancel{h}(-10x-5h)}{\cancel{h}\cdot x^2(x+h)^2} = \frac{-10}{x^3}$$

$$f'(a) = f'(-1) = \frac{-10}{(-1)^3} = \boxed{10}$$

3. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ โดยใช้สูตร

3.1) $f(x) = \boxed{x^2+5x-1}$

$f'(x) = 2x+5$

3.2) $f(x) = \boxed{4x^3-5}$

$f'(x) = 12x^2$

3.3) $f(x) = \boxed{x^7+3x^5+x-4}$

$f'(x) = 7x^6+15x^4+1$

3.4) $f(x) = \boxed{\frac{1}{5}x^4-\frac{1}{4}x^3+\frac{5}{3}x^2}$

$f'(x) = \frac{4}{5}x^3-\frac{3}{4}x^2+\frac{10}{3}x^1$

3.5) $f(x) = \boxed{\frac{1}{\sqrt{x}}-2\sqrt{x}-5x}$

$f'(x) = \left(x^{-1/2}-2x^{1/2}-5x\right)'$

$= -\frac{1}{2}x^{-3/2}-x^{-1/2}-5$

$f'(x) = -\frac{1}{2\sqrt{x^3}}-\frac{1}{\sqrt{x}}-5$

3.6) $f(x) = \boxed{\frac{1}{x^{1/4}}+\frac{1}{x^{4/3}}+x^{4/3}+x^{1/4}}$

$f'(x) = \left(x^{-1/4}+x^{-4/3}+x^{4/3}+x^{1/4}\right)'$

$= -\frac{1}{4}x^{-5/4}-\frac{4}{3}x^{-7/3}+\frac{4}{3}x^{1/3}+\frac{1}{4}x^{-3/4}$

$f'(x) = -\frac{1}{4x^{5/4}}-\frac{4}{3x^{7/3}}+\frac{4}{3}x^{1/3}+\frac{1}{4x^{3/4}}$

3.7) $f(x) = \boxed{(3x^2+5x+5)(5x-4)}$

$f'(x) = (3x^2+5x+5)\cdot(5x-4)'+(3x^2+5x+5)'\cdot(5x-4)$

$= 5(3x^2+5x+5)+(6x+5)(5x-4)$

$= 15x^2+25x+25+30x^2+x-20$

$f'(x) = 45x^2+26x+5$

3.8) $f(x) = \boxed{(\sqrt{x}-5)(x+1)}$

$f'(x) = (\sqrt{x}-5)\cdot(x+1)'+(\sqrt{x}-5)'\cdot(x+1)$

$= 1(\sqrt{x}-5)+\frac{1}{2\sqrt{x}}(x+1)$

$f'(x) = \frac{3x-10\sqrt{x}+1}{2\sqrt{x}}$

3.9) $f(x) = \boxed{\dfrac{5+3x}{x-2}}$

$$f'(x) = \frac{(5+3x)'(x-2)-(5+3x)\cdot(x-2)'}{(x-2)^2}$$

$$= \frac{3(x-2)-1(5+3x)}{(x-2)^2}$$

$$= \frac{\cancel{3x}-6-5-\cancel{3x}}{(x-2)^2}$$

$$\therefore f'(x) = \frac{-11}{(x-2)^2}$$

3.10) $f(x) = \boxed{\dfrac{4-x^5}{\sqrt{x}}}$

$$f'(x) = (4-x^5)(x^{-\frac{1}{2}})' + (4-x^5)'\cdot(x^{-\frac{1}{2}})$$

$$= -\frac{1}{2}x^{-3/2}(4-x^5) + (-5x^4)x^{-1/2}$$

$$= -2x^{-3/2} + \frac{1}{2}x^{7/2} - 5x^{7/2}$$

$$f'(x) = -2x^{-3/2} - \frac{9}{2}x^{7/2}$$

3.11) $f(x) = \boxed{\left(\dfrac{4}{x^2}+\dfrac{1}{x^5}\right)(2x^3+5)}$

$$f'(x) = \left(\frac{4}{x^2}+\frac{1}{x^5}\right)(2x^3+5)' + \left(\frac{4}{x^2}+\frac{1}{x^5}\right)'(2x^3+5)$$

$$= 6x^2\left(\frac{4}{x^2}+\frac{1}{x^5}\right) + \left(-\frac{8}{x^3}-\frac{5}{x^6}\right)(2x^3+5)$$

$$= 24 + \frac{6}{x^3} - 16 - \frac{50}{x^3} - \frac{25}{x^6}$$

$$f'(x) = 8 - \frac{44}{x^3} - \frac{25}{x^6}$$

3.12) $f(x) = \boxed{(5x+3x^4)\left(\dfrac{x+3}{x-4}\right)}$

$$f'(x) = (5x+3x^4)\left(\frac{x+3}{x-4}\right)' + (5x+3x^4)'\cdot\left(\frac{x+3}{x-4}\right)$$

$$= \frac{(5x+3x^4)(-7)}{(x-4)^2} + \frac{(5+12x^3)(x+3)}{(x-4)}$$

$$f'(x) = \frac{12x^5-33x^4-144x^3+5x^2-40x-60}{(x-4)^2}$$

4. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ ณ จุดที่กำหนดให้

4.1) $f(x) = \boxed{\dfrac{1}{\sqrt{x}}-2\sqrt{x}-5x}$, $a = \boxed{4}$

$$f'(x) = -\frac{1}{2}x^{-3/2} - x^{-1/2} - 5$$

$$= -\frac{1}{2x^{3/2}} - \frac{1}{\sqrt{x}} - 5$$

$$f'(a) = f'(4) = \frac{-1}{2(4)^{3/2}} - \frac{1}{\sqrt{4}} - 5$$

$$= -\frac{1}{16} - \frac{1}{2} - 5 = \boxed{\frac{-89}{16}}$$

4.2) $f(x) = \boxed{x^7+3x^5+x-4}$, $a = \boxed{2}$

$$f'(x) = 7x^6+15x^4+1$$

$$f'(a) = f'(2) = 7(2)^6+15(2)^4+1$$

$$= \boxed{689}$$

Diff02 for No.2

$$No01 = \begin{bmatrix} .1 = (f(x) = x^2 + 5x - 1) \\ .2 = (f(x) = 4x^3 - 5) \\ .3 = \left(f(x) = \dfrac{5}{x}\right) \\ .4 = (f(x) = 4x^{(1/3)}) \end{bmatrix}, \quad , No02 = \begin{bmatrix} .1 = [f(x) = 2x^3 + 5, a = 5] \\ .2 = \left[f(x) = \dfrac{5}{x^2}, a = -1\right] \end{bmatrix}$$

$$No03 = \begin{bmatrix} .1 = (f(x) = x^2 + 5x - 1) & .2 = (f(x) = 4x^3 - 5) \\ .3 = (f(x) = x^7 + 3x^5 + x - 4) & .4 = \left(f(x) = \dfrac{1}{5}x^4 - \dfrac{1}{4}x^3 + \dfrac{5}{3}x^2\right) \\ .5 = \left(f(x) = \dfrac{1}{\sqrt{x}} - 2\sqrt{x} - 5x\right) & .6 = \left(f(x) = \dfrac{1}{x^{(1/4)}} + \dfrac{1}{x^{(4/3)}} + x^{(4/3)} + x^{(1/4)}\right) \\ .7 = (f(x) = (3x^2 + 5x + 5)(5x - 4)) & .8 = (f(x) = (\sqrt{x} - 5)(x + 1)) \\ .9 = \left(f(x) = \dfrac{5 + 3x}{x - 2}\right) & .10 = \left(f(x) = \dfrac{4 - x^5}{\sqrt{x}}\right) \\ .11 = \left(f(x) = \left(\dfrac{4}{x^2} + \dfrac{1}{x^5}\right)(2x^3 + 5)\right) & .12 = \left(f(x) = (5x + 3x^4)\left[\dfrac{x + 3}{x - 4}\right]\right) \end{bmatrix}, \quad , \begin{bmatrix} \begin{bmatrix}\begin{bmatrix} P \\ V \\ S \\ S \end{bmatrix}\end{bmatrix} \\ \& \\ \begin{bmatrix}\begin{bmatrix} M \\ a \\ t \\ h \\ @ \\ M \\ U \\ T \end{bmatrix}\end{bmatrix} \end{bmatrix}$$

$$No04 = \begin{bmatrix} .1 = \left[f(x) = \dfrac{1}{\sqrt{x}} - 2\sqrt{x} - 5x, a = 4\right] \\ .2 = [f(x) = x^7 + 3x^5 + x - 4, a = 2] \end{bmatrix}$$

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Diff02 Answers for No.2

$$Ans1 = \begin{bmatrix} .1 = (f'(x) = 2x + 5) \\ .2 = (f'(x) = 12x^2) \\ .3 = \left(f'(x) = -\frac{5}{x^2}\right) \\ .4 = \left(f'(x) = \frac{4}{3x^{(2/3)}}\right) \end{bmatrix}, \quad , \; Ans2 = \begin{bmatrix} .1 = (f'(5) = 150) \\ .2 = (f'(-1) = 10) \end{bmatrix}, \quad , \; Ans4 = \begin{bmatrix} .1 = \left[f'(4) = \frac{-89}{16}\right] \\ .2 = [f'(2) = 689] \end{bmatrix}$$

$$Ans3 = \begin{bmatrix} .1 = (f'(x) = 2x + 5) & .2 = (f'(x) = 12x^2) \\ .3 = (f'(x) = 7x^6 + 15x^4 + 1) & .4 = \left(f'(x) = \frac{4}{5}x^3 - \frac{3}{4}x^2 + \frac{10}{3}x\right) \\ .5 = \left(f'(x) = -\frac{1}{2x^{(3/2)}} - \frac{1}{\sqrt{x}} - 5\right) & .6 = \left(f'(x) = -\frac{1}{4x^{(5/4)}} - \frac{4}{3x^{(7/3)}} + \frac{4x^{(1/3)}}{3} + \frac{1}{4x^{(3/4)}}\right) \\ .7 = (f'(x) = 26x + 5 + 45x^2) & .8 = \left(f'(x) = -\frac{-3x - 1 + 10\sqrt{x}}{2\sqrt{x}}\right) \\ .9 = \left(f'(x) = -\frac{11}{(x-2)^2}\right) & .10 = \left(f'(x) = -\frac{9x^5 + 4}{2x^{(3/2)}}\right) \\ .11 = \left(f'(x) = \frac{8x^6 - 44x^3 - 25}{x^6}\right) & .12 = \left(f'(x) = \frac{12x^5 - 33x^4 - 144x^3 + 5x^2 - 40x - 60}{(x-4)^2}\right) \end{bmatrix}, \quad , \; \begin{bmatrix} \underline{:)} \\ :(\\ \left[\begin{bmatrix} P \\ V \\ S \\ S \end{bmatrix}\right] \\ \& \\ \left[\begin{bmatrix} M \\ a \\ t \\ h \\ @ \\ M \\ U \\ T \end{bmatrix}\right] \\ \underline{:)} \\ :(\end{bmatrix}$$

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