



ชื่อ-นามสกุล

เลขประจำตัว

No. 2

แบบฝึกหัดเรื่อง ตรีโกณมิติ

1. จงหาค่าของ

1.1) $\sin\left(\frac{\pi}{12}\right) =$

$$\begin{aligned} \sin\frac{\pi}{12} &= \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \\ &= \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4} \\ &= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6}-\sqrt{2}}{4} \end{aligned}$$

1.2) $\cos(105^\circ) =$

$$\begin{aligned} \cos(105^\circ) &= \cos(60^\circ + 45^\circ) \\ &= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ \\ &= \frac{1}{2}\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2}-\sqrt{6}}{4} \end{aligned}$$

1.3) $\tan(75^\circ) =$

$$\begin{aligned} \tan 75^\circ &= \tan(45^\circ + 30^\circ) \\ &= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} \\ &= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1\left(\frac{1}{\sqrt{3}}\right)} \end{aligned}$$

1.4) $\cos(-165^\circ) = \cos(165^\circ)$

$$\begin{aligned} &= \cos(120^\circ + 45^\circ) \\ &= \cos(120^\circ)\cos(45^\circ) - \sin(120^\circ)\sin(45^\circ) \\ &= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} \end{aligned}$$

1.5) $\sin\left(\frac{23\pi}{12}\right) =$

$$\begin{aligned} \sin\frac{23\pi}{12} &= \sin\left(\frac{5\pi}{3} + \frac{\pi}{4}\right) \\ &= \sin\frac{5\pi}{3}\cos\frac{\pi}{4} + \cos\frac{5\pi}{3}\sin\frac{\pi}{4} \\ &= -\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{1}{2}\frac{\sqrt{2}}{2} \\ &= \frac{-\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

1.6) $\sec\left(-\frac{\pi}{12}\right) =$

$$\begin{aligned} \cos\frac{\pi}{12} &= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \\ &= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4} \\ &= \frac{1}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2} + \sqrt{6}}{4} \end{aligned} \quad \left| \begin{aligned} \sec\left(-\frac{\pi}{12}\right) &= \frac{1}{\cos\left(-\frac{\pi}{12}\right)} \\ &= \frac{1}{\cos\left(\frac{\pi}{12}\right)} \\ &= \frac{4}{\sqrt{2} + \sqrt{6}} \\ &= \sqrt{6} - \sqrt{2} \end{aligned} \right.$$

1.7) $\csc(195^\circ) =$

$$\begin{aligned} &= \sin(150^\circ + 45^\circ) \\ &= \sin 150^\circ \cos 45^\circ + \cos 150^\circ \sin 45^\circ \\ &= \frac{1}{2}\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2} - \sqrt{3}\sqrt{2}}{4} \end{aligned} \quad \left| \begin{aligned} \csc 195^\circ &= \frac{4}{\sqrt{2} - \sqrt{6}} \\ &= -\sqrt{2} - \sqrt{6} \end{aligned} \right.$$

1.8) $\cot(165^\circ) =$

$$\begin{aligned} \tan(165^\circ) &= \tan(120^\circ + 45^\circ) \\ &= \frac{\tan 120^\circ + \tan 45^\circ}{1 - \tan 120^\circ \tan 45^\circ} \\ &= \frac{-\sqrt{3} + 1}{1 - (-\sqrt{3})(1)} \\ &= \frac{1 - \sqrt{3}}{1 + \sqrt{3}} \end{aligned} \quad \left| \begin{aligned} \cot 165^\circ &= \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \\ &= -2 - \sqrt{3} \end{aligned} \right.$$

2. จงหาค่าของ

$$2.1) \boxed{2 \sin(15^\circ) \cos(15^\circ)} =$$

$$\leq \sin(2 \cdot 15^\circ)$$

$$\leq \sin(30^\circ)$$

$$\leq \frac{1}{2}$$

$$2.2) \boxed{\sin(35^\circ) \cos(5^\circ) - \cos(35^\circ) \sin(5^\circ)} =$$

$$\leq \sin(35^\circ - 5^\circ)$$

$$\leq \sin 30^\circ$$

$$\leq \frac{1}{2}$$

$$2.3) \boxed{\sin(10^\circ) \cos(35^\circ) + \cos(10^\circ) \sin(35^\circ)} =$$

$$\leq \sin(10^\circ + 35^\circ)$$

$$\leq \sin 45^\circ$$

$$\leq \frac{\sqrt{2}}{2}$$

$$2.4) \boxed{\cos(22.5^\circ)^2 - \sin(22.5^\circ)^2} =$$

$$\leq \cos(2 \cdot 22.5^\circ)$$

$$\leq \cos 45^\circ$$

$$\leq \frac{\sqrt{2}}{2}$$

$$2.5) \boxed{2 \cos\left(\frac{\pi}{12}\right)^2 - 1} =$$

$$= \cos\left(2 \cdot \frac{\pi}{12}\right)$$

$$\leq \cos\left(\frac{\pi}{6}\right)$$

$$\leq \frac{\sqrt{3}}{2}$$

$$2.6) \boxed{\frac{\tan\left(\frac{7\pi}{36}\right) + \tan\left(\frac{5\pi}{36}\right)}{1 - \tan\left(\frac{7\pi}{36}\right) \tan\left(\frac{5\pi}{36}\right)}} =$$

$$\leq \tan\left(\frac{7\pi}{36} + \frac{5\pi}{36}\right)$$

$$\leq \tan\left(\frac{\pi}{3}\right)$$

$$\leq \tan \frac{\pi}{3}$$

$$\leq \sqrt{3}$$

$$2.7) \boxed{\frac{\tan\left(\frac{2\pi}{5}\right) - \tan\left(\frac{\pi}{15}\right)}{1 + \tan\left(\frac{2\pi}{5}\right) \tan\left(\frac{\pi}{15}\right)}} =$$

$$\leq \tan\left(\frac{2\pi}{5} - \frac{\pi}{15}\right)$$

$$\leq \tan \frac{\pi}{3}$$

$$\leq \sqrt{3}$$

$$2.8) \boxed{\frac{2 \tan(22.5^\circ)}{1 - \tan(22.5^\circ)^2}} =$$

$$\leq \tan(2 \cdot 22.5^\circ)$$

$$\leq \tan 45^\circ$$

$$\leq 1$$

$$2.9) \boxed{1 - 2 \sin\left(\frac{\pi}{8}\right)^2} =$$

$$\leq \cos\left(2 \cdot \frac{\pi}{8}\right)$$

$$\leq \cos \frac{\pi}{4}$$

$$\leq \frac{\sqrt{2}}{2}$$

$$2.10) \boxed{\cos\left(\frac{\pi}{18}\right) \cos\left(\frac{\pi}{9}\right) - \sin\left(\frac{\pi}{18}\right) \sin\left(\frac{\pi}{9}\right)} =$$

$$\leq \cos\left(\frac{\pi}{18} + \frac{\pi}{9}\right)$$

$$\leq \cos \frac{\pi}{6}$$

$$\leq \frac{\sqrt{3}}{2}$$

3. กำหนด $A = 0$, $B = \frac{\pi}{2}$, $C = \pi$, $D = \frac{3\pi}{2}$

ถ้า $\text{Con1} = \sin(\alpha) = \frac{1}{2}$, $A \leq \alpha \leq B$ และ $\text{Con2} = \cos(\beta) = -\frac{5}{6}$, $C \leq \beta \leq D$ จงหา

Q3

3.1) $\cos(\alpha) =$

$$= \sqrt{1 - \sin^2(\alpha)}$$

$$= \sqrt{1 - \frac{1}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

3.2) $\sin(\beta) =$

$$= -\sqrt{1 - \cos^2(\beta)}$$

$$= -\sqrt{1 - \frac{25}{36}}$$

$$= -\sqrt{\frac{11}{36}}$$

$$= -\frac{\sqrt{11}}{6}$$

3.3) $\tan(\beta) =$

$$= \frac{\sin(\beta)}{\cos(\beta)}$$

$$= \frac{-\frac{\sqrt{11}}{6}}{-\frac{5}{6}}$$

$$= \frac{\sqrt{11}}{5}$$

3.4) $\sin(\beta - \alpha) =$

$$= \sin\beta \cos\alpha - \cos\beta \sin\alpha$$

$$= \frac{\sqrt{11}}{6} \cdot \frac{\sqrt{3}}{2} - \left(-\frac{5}{6}\right) \left(\frac{1}{2}\right)$$

$$= \frac{-\sqrt{11}\sqrt{3} + 5}{12}$$

3.5) $\cos(\alpha + \beta) =$

$$= \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$= \left(\frac{\sqrt{3}}{2}\right) \left(-\frac{5}{6}\right) - \left(\frac{1}{2}\right) \left(-\frac{\sqrt{11}}{6}\right)$$

$$= \frac{-5\sqrt{3} + \sqrt{11}}{12}$$

3.6) $\tan(\alpha - \beta) =$

$$= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta} = \frac{5\sqrt{3} - 3\sqrt{11}}{15} \cdot \frac{15}{15 + \sqrt{3}\sqrt{11}}$$

$$= \frac{\frac{\sqrt{3}}{3} - \frac{\sqrt{11}}{5}}{1 + \frac{\sqrt{3}}{3} \cdot \frac{\sqrt{11}}{5}}$$

$$= \frac{5\sqrt{3} - 3\sqrt{11}}{15 + \sqrt{3}\sqrt{11}}$$

3.7) $\cos(2\alpha) =$

$$= 2\cos^2\alpha - 1$$

$$= 2 \cdot \frac{3}{4} - 1$$

$$= \frac{3}{2} - 1$$

$$= \frac{1}{2}$$

3.8) $\tan(2\beta) =$

$$= \frac{2\tan\beta}{1 - \tan^2\beta} = \frac{2\sqrt{11}}{5} \cdot \frac{25}{14}$$

$$= \frac{2 \cdot \frac{\sqrt{11}}{5}}{1 - \frac{11}{25}}$$

$$= \frac{2\sqrt{11}}{5} \cdot \frac{25}{14}$$

$$= \frac{5\sqrt{11}}{7}$$

4. กำหนด $A = \frac{3\pi}{2}$, $B = 2\pi$, $C = \frac{\pi}{2}$, $D = \pi$

ถ้า $Con1 = \sin(\alpha) = -\frac{4}{5}$, $A \leq \alpha \leq B$ และ $Con2 = \cos(\beta) = -\frac{1}{4}$, $C \leq \beta \leq D$ จงหา

Q2

4.1) $\cos(\alpha) =$

$\sqrt{1 - \sin^2 \alpha}$

$\sqrt{1 - \frac{16}{25}}$

$\frac{3}{5}$

4.2) $\sin(\beta) =$

$\sqrt{1 - \cos^2 \beta}$

$\sqrt{1 - \frac{1}{16}}$

$\frac{\sqrt{15}}{4}$

4.3) $\tan(\beta) =$

$\frac{\frac{\sqrt{15}}{4}}{-\frac{1}{4}}$

$-\sqrt{15}$

4.4) $\sin(\alpha - \beta) =$

$\sin \alpha \cos \beta - \cos \alpha \sin \beta$

$(-\frac{4}{5})(-\frac{1}{4}) - (\frac{3}{5})(\frac{\sqrt{15}}{4})$

$\frac{1}{5} - \frac{3\sqrt{15}}{20}$

4.5) $\cos(\beta - \alpha) =$

$\cos \beta \cos \alpha + \sin \beta \sin \alpha$

$(-\frac{1}{4})(\frac{3}{5}) + (\frac{\sqrt{15}}{4})(-\frac{4}{5})$

$-\frac{3}{20} - \frac{\sqrt{15}}{5}$

4.6) $\tan(\alpha + \beta) =$

$\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{-\frac{4}{5} - \sqrt{15}}{1 - (-\frac{4}{5})(-\sqrt{15})}$

$\frac{-\frac{4}{5} - \sqrt{15}}{1 - \frac{4\sqrt{15}}{5}}$

$\frac{3}{3 - 4\sqrt{15}}$

$\frac{-4 - 3\sqrt{15}}{3 - 4\sqrt{15}}$

4.7) $\sin(2\beta) =$

$2 \sin \beta \cos \beta$

$2(\frac{\sqrt{15}}{4})(-\frac{1}{4})$

$-\frac{\sqrt{15}}{8}$

4.8) $\tan(2\alpha) =$

$\frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{-\frac{8}{3}}{1 - \frac{64}{9}}$

$\frac{2(-\frac{4}{3})}{1 - \frac{16}{9}} = \frac{-\frac{8}{3}}{\frac{2}{9}} = -\frac{8}{3} \cdot \frac{9}{2} = -12$

$\frac{72}{21} = \frac{24}{7}$

5. ถ้า $Cond = \cos(32^\circ) = 0.848$

จงหา $Question = \cos(16^\circ)$

$\sqrt{\frac{1 + \cos 32^\circ}{2}} = \sqrt{\frac{1.848}{2}}$

$= \sqrt{0.924}$

$= \frac{1 + 0.848}{2} = 0.961$

6. ถ้า $Cond = \sin(54^\circ) = 0.809$

จงหา $Question = \sin(27^\circ)$

$\cos 54^\circ = \sqrt{1 - 0.654} = 0.588$

$\sin 27^\circ = \sqrt{\frac{1 - \cos 54^\circ}{2}} = \sqrt{\frac{0.412}{2}}$

$= \sqrt{0.206} = \frac{(1 - 0.588)/2}{2} = 0.454$

