



แบบฝึกหัดเรื่อง เมทริกซ์

ชื่อ-นามสกุล

เลขประจำตัว No.3

Determinant

กำหนดเมทริกซ์ **A - F** และค่าคงที่ **m, p** จงหา

1. $\det(A) = \det([17]) = 17$

2. $\det(B) = \det([-13]) = -13$

3. $\det(C) = \begin{vmatrix} 7 & -6 \\ 4 & -2 \end{vmatrix} = -14 + 24 = 10$

4. $\det(D) = \begin{vmatrix} -8 & 10 \\ 10 & -6 \end{vmatrix} = 48 - 100 = -52$

5. $\det(E) = \begin{vmatrix} 0 & -2 & 4 & 0 & -2 \\ 2 & -2 & 3 & 2 & -2 \\ 2 & -4 & 3 & 2 & -4 \end{vmatrix}$
 $= 0 - 12 - 32 - (-16 + 0 - 12)$
 $= -16$

6. $\det(F) = \begin{vmatrix} 3 & 3 & 2 & 3 & 3 \\ 0 & 3 & 4 & 0 & 3 \\ -2 & 4 & 3 & -2 & 4 \end{vmatrix}$
 $= 27 - 24 + 0 - (-12 + 48 + 0)$
 $= -33$

7. $m \det(C) - \det(pE) = 4(10) - 3^3(-16)$

8. $\det(mD) - p \det(F) = 4^2(-52) - 3(-33)$

$m = 4$
 $p = 3$
 $= 472$

$= -733$

$\det(kA) = k^n \cdot \det(A)$

จงแก้สมการต่อไปนี้

9. $\det \begin{pmatrix} -2 & 2 \\ x & 4 \end{pmatrix} = -4$ จงหาค่า x

$-8 - 2x = -4$
 $-2x = 4$
 $x = -2$

10. $\det \begin{pmatrix} 4 & y & -2 \\ -4 & 4 & 2 \\ -3 & -2 & 3 \end{pmatrix} = 12$ จงหาค่า y

$\begin{vmatrix} 4 & y & -2 \\ -4 & 4 & 2 \\ -3 & -2 & 3 \end{vmatrix} = 12$
 $48 - 6y - 16 - (24 - 16 - 12y) = 12$

$24 + 6y = 12$
 $6y = -12$
 $y = -2$

Inverse

11. กำหนดเมทริกซ์ **G** จงแสดงการหา G^{-1} $G = \begin{bmatrix} 3 & 5 \\ 2 & 5 \end{bmatrix}$

$G^{-1} = \frac{1}{15-10} \begin{bmatrix} 5 & -5 \\ -2 & 3 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 5 & -5 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -\frac{2}{5} & \frac{3}{5} \end{bmatrix}$

Cramer's rule

จงใช้กฎของคราเมอร์หาผลเฉลยของระบบสมการต่อไปนี้

12. System1=

$$\begin{cases} 5x - 4y = -16 \\ 2x + 4y = -40 \end{cases}$$

9_u $A = \begin{bmatrix} 5 & -4 \\ 2 & 4 \end{bmatrix}$ $D = \begin{bmatrix} -16 \\ -40 \end{bmatrix}$

det A = 28

$$x = \frac{\begin{vmatrix} -16 & -4 \\ -40 & 4 \end{vmatrix}}{28} = \frac{-64 - 160}{28} = -8$$

$$y = \frac{\begin{vmatrix} 5 & -16 \\ 2 & -40 \end{vmatrix}}{28} = \frac{-200 + 32}{28} = -6$$

13. System2=

$$\begin{cases} -2x - y = -8 \\ -x + y = -7 \end{cases}$$

9_u $A = \begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} -8 \\ -7 \end{bmatrix}$

det A = -3

$$x = \frac{\begin{vmatrix} -8 & -1 \\ -7 & 1 \end{vmatrix}}{-3} = \frac{-8 - 7}{-3} = 5$$

$$y = \frac{\begin{vmatrix} -2 & -8 \\ -1 & -7 \end{vmatrix}}{-3} = \frac{14 - 8}{-3} = -2$$

14. System3=

$$\begin{cases} x - y = 1 \\ 2y + z = -15 \\ x + z = -8 \end{cases}$$

$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 1 \\ -15 \\ -8 \end{bmatrix}$

det A = $\begin{vmatrix} 1 & -1 & 0 & 1 & -1 \\ 0 & 2 & 1 & 0 & 2 \\ 1 & 0 & 1 & 1 & 0 \end{vmatrix} = 1$

$$x = \frac{\begin{vmatrix} 1 & -1 & 0 & 1 & -1 \\ -15 & 2 & 1 & 0 & 2 \\ -8 & 0 & 1 & 1 & 0 \end{vmatrix}}{1} = -5$$

$$y = \frac{\begin{vmatrix} 1 & 1 & 0 & 1 & -1 \\ 0 & -15 & 1 & 0 & -15 \\ 1 & -8 & 1 & 1 & -8 \end{vmatrix}}{1} = -6$$

$$z = \frac{\begin{vmatrix} 1 & -1 & 1 & 1 & -1 \\ 0 & 2 & -15 & 0 & 2 \\ 1 & 0 & -8 & 1 & 0 \end{vmatrix}}{1} = -3$$

15. System4=

$$\begin{cases} 3y + 3z = 3 \\ x - 3z = -17 \\ -x - y = 10 \end{cases}$$

$A = \begin{bmatrix} 0 & 3 & 3 \\ 1 & 0 & -3 \\ -1 & -1 & 0 \end{bmatrix}$ $D = \begin{bmatrix} 3 \\ -17 \\ 10 \end{bmatrix}$

det A = $\begin{vmatrix} 0 & 3 & 3 & 0 & 3 \\ 1 & 0 & -3 & 1 & 0 \\ -1 & -1 & 0 & -1 & -1 \end{vmatrix} = 6$

$$x = \frac{\begin{vmatrix} 3 & 3 & 3 & 3 & 3 \\ -17 & 0 & -3 & -17 & 0 \\ 10 & -1 & 0 & 10 & -1 \end{vmatrix}}{6} = \frac{-48}{6} = -8$$

$$y = \frac{\begin{vmatrix} 0 & 3 & 3 & 0 & 3 \\ 1 & -17 & -3 & 1 & -17 \\ -1 & 10 & 0 & -1 & 10 \end{vmatrix}}{6} = \frac{-12}{6} = -2$$

$$z = \frac{\begin{vmatrix} 0 & 3 & 3 & 0 & 3 \\ 1 & 0 & -17 & 1 & 0 \\ -1 & -1 & 10 & -1 & -1 \end{vmatrix}}{6} = \frac{18}{6} = 3$$

