



ชื่อ-นามสกุล

เลขประจำตัว

No.3

แบบฝึกหัดเรื่อง อนุพันธ์

1. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้

1.1) $f(x) = (5x+4)^7$

$$f'(x) = 7(5x+4)^6 \cdot (5)$$

$$= \underline{35(5x+4)^6}$$

1.2) $f(x) = (2x^4 + 2x + 7)^7$

$$f'(x) = 7(2x^4 + 2x + 7) \cdot (8x^3 + 2)$$

$$= \underline{7(8x^3 + 2)(2x^4 + 2x + 7)}$$

1.3) $f(x) = \sqrt{3-2x}$

$$f'(x) = \frac{1}{2}(3-2x)^{\frac{1}{2}-1} \cdot (-2)$$

$$= \frac{1}{2\sqrt{3-2x}} \cdot (-2) = \underline{\frac{-1}{\sqrt{3-2x}}}$$

1.4) $f(x) = \frac{1}{5x^5+4} = (5x^5+4)^{-1}$

$$f'(x) = -1(5x^5+4)^{-2} \cdot (25x^4)$$

$$= -25x^4(5x^5+4)^{-2}$$

$$= \underline{\frac{-25x^4}{(5x^5+4)^2}}$$

1.5) $f(x) = \frac{1}{(2x^2+2x+5)^7}$

$$f'(x) = -7(2x^2+2x+5)^{-8} \cdot (4x+2)$$

$$= \underline{\frac{-7(4x+2)}{(2x^2+2x+5)^8}}$$

1.6) $f(x) = \frac{1}{\sqrt{5-4x}} = (5-4x)^{-1/2}$

$$f'(x) = -\frac{1}{2}(5-4x)^{-\frac{1}{2}-1} \cdot (-4)$$

$$= 2(5-4x)^{-3/2}$$

$$= \underline{\frac{2}{(5-4x)^{3/2}}}$$

1.7) $f(x) = (2x+3)^3(5x-6)$

$$f'(x) = 5(2x+3)^3 + 3(2x+3)^2 \cdot (2)(5x-6)$$

$$= \underline{5(2x+3)^3 + 6(5x-6)(2x+3)^2}$$

1.8) $f(x) = \frac{(2x+3)^3}{(5x-6)^3}$

$$f'(x) = \frac{3(2x+3)^2 \cdot (2)(5x-6)^3 - 3(5x-6)^2 \cdot (5)(2x+3)^3}{((5x-6)^3)^2}$$

$$= \frac{(5x-6)^2 [6(2x+3)^2(5x-6) - 15(2x+3)^3]}{(5x-6)^6}$$

$$= \underline{\frac{(2x+3)^2 [6(5x-6) - 15(2x+3)]}{(5x-6)^4} = \frac{-81(2x+3)^2}{(5x-6)^4}}$$

2. กำหนดข้อมูลตามตารางด้านล่าง จงหา $F'(k=2)$ เมื่อ

| x | f(x) | f'(x) | g(x) | g'(x) |
|----|------|-------|------|-------|
| -6 | 3 | -4 | -3 | 16 |
| -5 | 4 | -3 | -4 | 14 |
| -4 | 5 | -2 | -5 | 12 |
| -3 | 6 | -1 | -6 | 10 |
| -2 | 7 | 0 | -7 | 8 |
| -1 | -6 | 1 | 6 | 6 |
| 0 | -5 | 2 | 5 | 4 |

| x | f(x) | f'(x) | g(x) | g'(x) |
|---|------|-------|------|-------|
| 1 | -4 | 3 | 4 | 2 |
| 2 | -3 | 4 | 3 | 0 |
| 3 | -2 | 5 | 2 | -2 |
| 4 | -1 | 6 | 1 | -4 |
| 5 | 0 | 7 | 0 | -6 |
| 6 | 1 | 8 | -1 | -8 |
| 7 | 2 | 9 | -2 | -10 |

2.1) $F(x) = f(g(x))$
 $F'(k) = f'(g(k)) \cdot g'(k)$
 $F'(2) = f'(g(2)) \cdot g'(2)$
 $= f'(3) \cdot (0) = 5(0)$
 $F'(2) = \boxed{0}$

2.2) $F(x) = g(f(x))$
 $F'(k) = g'(f(k)) \cdot f'(k)$
 $F'(2) = g'(f(2)) \cdot f'(2)$
 $= g'(-3)(4) = 10(4)$
 $F'(2) = \boxed{40}$

3. กำหนดข้อมูลตามตารางด้านล่าง และ $F(x) = f(g(x))$ จงหา $F'(k=3)$ เมื่อ

| | | | | | | | | | |
|-------|----|----|----|----|----|----|----|----|---|
| x | -8 | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 |
| g(x) | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| g'(x) | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | 1 |

| | | | | | | | | | |
|-------|---|---|---|---|---|---|----|----|----|
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| g(x) | 4 | 5 | 6 | 7 | 8 | 9 | -8 | -7 | -6 |
| g'(x) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

3.1) $f(x) = x \cdot g(x)$
 $F'(k) = f'(g(k)) = (g(k) \cdot g'(g(k)))'$
 $= g(k)(g'(g(k)))' + g'(k) \cdot g'(g(k))$
 $F'(k) = g(k) \cdot g'(g(k)) \cdot g'(k) + g'(k) \cdot g'(g(k))$
 $F'(3) = g(3) \cdot g'(g(3)) \cdot g'(3) + g'(3) \cdot g'(g(3))$
 $= 6(7)(4) + (4)(9) = \boxed{204}$

3.2) $f(x) = \frac{g(x)}{x}$
 $F'(k) = f'(g(k)) = \left(\frac{g(g(k))}{g(k)} \right)'$
 $F'(k) = \frac{g'(g(k)) \cdot g'(k) \cdot g(k) - g(g(k)) \cdot g'(k)}{(g(k))^2}$
 $F'(3) = \frac{g'(g(3)) \cdot g'(3) \cdot g(3) - g(g(3)) \cdot g'(3)}{(g(3))^2} = \frac{7(4)(6) - 9(4)}{36} = \frac{11}{3}$

4. กำหนด $x(t) = \frac{4t^2}{5+t^2}$, $P(x) = 8\sqrt{x} - 9$ $k = 6$ จงหา

4.1) $\frac{dx}{dt} = \frac{(4t^2)'(5+t^2) - 4t^2(5+t^2)'}{(5+t^2)^2}$
 $= \frac{8t(5+t^2) - 4t^2(2t)}{(5+t^2)^2}$
 $\frac{dx}{dt} = \boxed{\frac{40t}{(5+t^2)^2}}$

4.2) $\frac{dP}{dx} = 8\left(\frac{1}{2}x^{\frac{1}{2}-1}\right)$
 $= 4x^{-1/2}$
 $\frac{dP}{dx} = \boxed{\frac{4}{\sqrt{x}}}$

4.3) $\frac{dP}{dt} \Big|_{t=k} = \frac{dP}{dx} \cdot \frac{dx}{dt} \Big|_{t=6}$
 $= \frac{4}{\sqrt{x}} \cdot \frac{40t}{(5+t^2)^2} = \frac{160t}{\sqrt{x}(5+t^2)^2}$
 $= \frac{160(6)}{\sqrt{\frac{4t^2}{5+t^2}} \cdot (5+t^2)^2} = \frac{160(6)}{\sqrt{\frac{4(6)^2}{5+36}} \cdot (41)^2}$
 $= \frac{960(41)}{12\sqrt{41}(41)^2} = \frac{80}{41\sqrt{41}} = \boxed{0.305}$

