แบบฝึกหัดเรื่อง อนุพันธ์

ชื่อ-นามสกุล Diff 02

เลขประจำตัว No. 03

1. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ โดยใช้นิยามของอนุพันธ์ $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

1.1) $f(x) = \boxed{5x^2-3x+4}$

$$f'(x) = \lim_{h\to 0}\frac{5(x+h)^2-3(x+h)+4-5x^2+3x-4}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{5x^2}+10xh+5h^2-\cancel{3x}-3h+\cancel{4}-\cancel{5x^2}+\cancel{3x}-\cancel{4}}{h}$$

$$= \lim_{h\to 0}\frac{5h^2+10xh-3h}{h}$$

$$= \lim_{h\to 0} 5h+10x-3 = 10x-3$$

1.2) $f(x) = \boxed{3x^3+4}$

$$f'(x) = \lim_{h\to 0}\frac{3(x+h)^3+4-3x^3-4}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{3x^3}+9x^2h+9xh^2+3h^3+\cancel{4}-\cancel{3x^3}-\cancel{4}}{h}$$

$$= \lim_{h\to 0} 9x^2+9xh+3h^2$$

$$= 9x^2$$

1.3) $f(x) = \boxed{\frac{6}{x}}$

$$f'(x) = \lim_{h\to 0}\frac{\frac{6}{x+h}-\frac{6}{x}}{h}$$

$$= \lim_{h\to 0}\frac{6x-6x-6h}{h(x^2+xh)}$$

$$= \lim_{h\to 0} -\frac{6}{x^2+xh}$$

$$= -\frac{6}{x^2}$$

1.4) $f(x) = \boxed{2\sqrt{x}}$

$$f'(x) = \lim_{h\to 0}\frac{2\sqrt{x+h}-2\sqrt{x}}{h}$$

$$= \lim_{h\to 0}\frac{4(x+h)-4x}{h(2\sqrt{x+h}+2\sqrt{x})}$$

$$= \lim_{h\to 0}\frac{4}{2\sqrt{x+h}+2\sqrt{x}}$$

$$= \frac{4}{4\sqrt{x}} = \frac{1}{\sqrt{x}}$$

2. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ ณ จุดที่กำหนดให้ โดยใช้นิยามของอนุพันธ์ $f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

2.1) $f(x) = \boxed{5x^3-1}$, $a = \boxed{3}$

$$f'(a) = \lim_{x\to 3}\frac{5x^3-1-134}{x-3}$$

$$= \lim_{x\to 3}\frac{5x^3-135}{x-3}$$

$$= \lim_{x\to 3}\frac{5(x^3-27)}{x-3}$$

$$= \lim_{x\to 3} 5(x^2+3x+9)$$

$$= 5(27) = 135$$

2.2) $f(x) = \boxed{\frac{6}{x^2}}$, $a = \boxed{-3}$

$$f'(a) = \lim_{x\to -3}\frac{\frac{6}{x^2}-\frac{6}{(-3)^2}}{x-(-3)}$$

$$= \lim_{x\to -3}\frac{\frac{6}{x^2}-\frac{2}{3}}{x+3}$$

$$= \lim_{x\to -3}\frac{18-2x^2}{3x^2(x+3)}$$

$$= \lim_{x\to -3}\frac{2(9-x^2)}{3x^2(x+3)}$$

$$= \lim_{x\to -3}\frac{2(3-x)}{3x^2}$$

$$= \frac{2(3-(-3))}{3(-3)^2}$$

$$= \frac{4}{9}$$

3. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ โดยใช้สูตร

3.1) $f(x) = \boxed{5x^2 - 3x + 4}$

$f'(x) = 5 \cdot 2x - 3 \cdot 1$

$= 10x - 3$

3.2) $f(x) = \boxed{3x^3 + 4}$

$f'(x) = 3 \cdot 3x^2$

$= 9x^2$

3.3) $f(x) = \boxed{5x^6 + 4x^5 + 5x^4 - x}$

$f'(x) = 5 \cdot 6x^5 + 4 \cdot 5x^4 + 5 \cdot 4x^3 - 1$

$= 30x^5 + 20x^4 + 20x^3 - 1$

3.4) $f(x) = \boxed{\frac{3}{4}x^4 - \frac{2}{3}x^3 + \frac{5}{6}x}$

$f'(x) = \frac{3}{4} \cdot 4x^3 - \frac{2}{3} \cdot 3x^2 + \frac{5}{6}$

$= 3x^3 - 2x^2 + \frac{5}{6}$

3.5) $f(x) = \boxed{\frac{4}{\sqrt{x}} + 2\sqrt{x} - 3x}$

$f'(x) = 4 \cdot \left(-\frac{1}{2}\right) x^{-\frac{3}{2}} + 2 \cdot \frac{1}{2} x^{-\frac{1}{2}} - 3$

$= -\frac{2}{x^{3/2}} + \frac{1}{x^{\frac{1}{2}}} - 3$

3.6) $f(x) = \boxed{\frac{1}{x^{4/5}} + x^{4/5} + x^{1/5} + \frac{1}{x^{5/4}}}$

$f'(x) = 1 \cdot \left(-\frac{4}{5}\right) x^{-\frac{9}{5}} + 1 \cdot \frac{4}{5} x^{-\frac{1}{5}} + \frac{1}{5} x^{-\frac{4}{5}} + \left(-\frac{5}{4}\right) x^{-\frac{9}{4}}$

$= -\frac{4}{5x^{9/5}} + \frac{4}{5x^{1/5}} + \frac{1}{5x^{4/5}} - \frac{5}{4x^{9/4}}$

3.7) $f(x) = \boxed{(5x^2 + 2x + 1)(3x - 4) = 15x^3 - 14x^2 - 5x - 4}$

$f'(x) = 15 \cdot 3x^2 - 14 \cdot 2x - 5$

$= 45x^2 - 28x - 5$

3.8) $f(x) = \boxed{(x + 5)(\sqrt{x} + 1) = x^{3/2} + x} + 5x^{\frac{1}{2}} + 5$

$f'(x) = \frac{3}{2} x^{\frac{1}{2}} + 1 + 5 \cdot \frac{1}{2} x^{-\frac{1}{2}}$

$= \frac{3\sqrt{x}}{2} + \frac{5}{2\sqrt{x}} + 1$

3.9) $f(x) = \boxed{\dfrac{3x+4}{4x-3}}$

$$f'(x) = \frac{(4x-3)(3x+4)' - (3x+4)(4x-3)'}{(4x-3)^2}$$

$$= \frac{(4x-3)(3) - (3x+4)(4)}{(4x-3)^2}$$

$$= \frac{12x-9-12x-16}{(4x-3)^2}$$

$$= -\frac{25}{(4x-3)^2}$$

3.10) $f(x) = \boxed{\dfrac{5x^5-x}{\sqrt{x}} = 5x^{\frac{9}{2}} - x^{\frac{1}{2}}}$

$$f'(x) = 5\cdot\frac{9}{2}x^{7/2} - \frac{1}{2}x^{-\frac{1}{2}}$$

$$= \frac{45}{2}x^{\frac{7}{2}} - \frac{1}{2x^{\frac{1}{2}}}$$

$$= \frac{45}{2}x^{\frac{7}{2}} \cdot \frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} - \frac{1}{2x^{\frac{1}{2}}}$$

$$= \frac{45x^4-1}{2\sqrt{x}}$$

3.11) $f(x) = \boxed{\left(\dfrac{3}{x^5} + \dfrac{4}{x^3}\right)(2x^4+5)}$

$$f(x) = \left(\frac{3+4x^2}{x^5}\right)(2x^4+5)$$

$$= (8x^6+6x^4+20x^2+15)/x^5$$

$$= 8x + 6x^{-1} + 20x^{-3} + 15x^{-5}$$

$$f'(x) = 8 + 6(-1)x^{-2} + 20(-3)x^{-4} + 15(-5)x^{-6}$$

$$= \frac{8x^6 - 6x^4 - 60x^2 - 75}{x^6}$$

3.12) $f(x) = \boxed{(4x^4+5x^3)\left(\dfrac{x-1}{x+2}\right)} = \dfrac{4x^5-4x^4+5x^4-5x^3}{x+2}$

$$= \frac{4x^5+x^4-5x^3}{x+2}$$

$$f'(x) = \frac{(x+2)(20x^4+4x^3-15x^2) - 4x^5 - x^4 + 5x^3}{(x+2)^2}$$

$$= \frac{20x^5+44x^4-7x^3-30x^2-4x^5-x^4+5x^3}{(x+2)^2}$$

$$= \frac{16x^5+43x^4-2x^3-30x^2}{(x+2)^2}$$

4. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ ณ จุดที่กำหนดให้

4.1) $f(x) = \boxed{\dfrac{4}{\sqrt{x}} + 2\sqrt{x} - 3x}$, $a = \boxed{4}$

$$f'(x) = 4\left(-\frac{1}{2}\right)x^{-\frac{3}{2}} + 2\left(\frac{1}{2}\right)x^{-\frac{1}{2}} - 3$$

$$= -\frac{2}{x\sqrt{x}} + \frac{1}{\sqrt{x}} - 3$$

$$f'(a) = -\frac{2}{4\sqrt{4}} + \frac{1}{\sqrt{4}} - 3$$

$$= -\frac{1}{4} + \frac{1}{2} - 3$$

$$= -\frac{11}{4}$$

4.2) $f(x) = \boxed{5x^6+4x^5+5x^4-x}$, $a = \boxed{-2}$

$$f'(x) = 5\cdot 6x^5 + 4\cdot 5x^4 + 5\cdot 4x^3 - 1$$

$$= 30x^5 + 20x^4 + 20x^3 - 1$$

$$f'(a) = 30(-2)^5 + 20(-2)^4 + 20(-2)^3 - 1$$

$$= -960 + 320 - 160 - 1$$

$$= -801$$

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
X Math@MUT XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX6300206-00003XX

Diff02 for No.3

$$No01 = \begin{bmatrix} .1 = (f(x) = 5x^2 - 3x + 4) \\ .2 = (f(x) = 3x^3 + 4) \\ .3 = \left(f(x) = \frac{6}{x}\right) \\ .4 = (f(x) = 2\sqrt{x}) \end{bmatrix}, \quad , No02 = \begin{bmatrix} .1 = [f(x) = 5x^3 - 1, a = 3] \\ .2 = \left[f(x) = \frac{6}{x^2}, a = -3\right] \end{bmatrix}$$

$$No03 = \begin{bmatrix} .1 = (f(x) = 5x^2 - 3x + 4) & .2 = (f(x) = 3x^3 + 4) \\ .3 = (f(x) = 5x^6 + 4x^5 + 5x^4 - x) & .4 = \left(f(x) = \frac{3}{4}x^4 - \frac{2}{3}x^3 + \frac{5}{6}x\right) \\ .5 = \left(f(x) = \frac{4}{\sqrt{x}} + 2\sqrt{x} - 3x\right) & .6 = \left(f(x) = \frac{1}{x^{(4/5)}} + x^{(4/5)} + x^{(1/5)} + \frac{1}{x^{(5/4)}}\right) \\ .7 = (f(x) = (5x^2 + 2x + 1)(-4 + 3x)) & .8 = (f(x) = (x + 5)(\sqrt{x} + 1)) \\ .9 = \left(f(x) = \frac{4 + 3x}{4x - 3}\right) & .10 = \left(f(x) = \frac{5x^5 - x}{\sqrt{x}}\right) \\ .11 = \left(f(x) = \left(\frac{3}{x^5} + \frac{4}{x^3}\right)(2x^4 + 5)\right) & .12 = \left(f(x) = (4x^4 + 5x^3)\left[\frac{x - 1}{x + 2}\right]\right) \end{bmatrix}, \quad , \begin{bmatrix} \begin{bmatrix}\begin{bmatrix} P \\ V \\ S \\ S \end{bmatrix}\end{bmatrix} \\ \& \\ \begin{bmatrix}\begin{bmatrix} M \\ a \\ t \\ h \\ @ \\ M \\ U \\ T \end{bmatrix}\end{bmatrix} \end{bmatrix}$$

$$No04 = \begin{bmatrix} .1 = \left[f(x) = \frac{4}{\sqrt{x}} + 2\sqrt{x} - 3x, a = 4\right] \\ .2 = [f(x) = 5x^6 + 4x^5 + 5x^4 - x, a = -2] \end{bmatrix}$$

X [Page = 0003] XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

:
:
:
:
:
:

Diff02 Answers for No.3

$$Ans1 = \begin{bmatrix} .1 = (f'(x) = 10\,x - 3) \\ .2 = (f'(x) = 9\,x^2) \\ .3 = \left(f'(x) = -\dfrac{6}{x^2}\right) \\ .4 = \left(f'(x) = \dfrac{1}{\sqrt{x}}\right) \end{bmatrix}, \quad , \quad Ans2 = \begin{bmatrix} .1 = (f'(3) = 135) \\ .2 = \left(f'(-3) = \dfrac{4}{9}\right) \end{bmatrix}, \quad , \quad Ans4 = \begin{bmatrix} .1 = \left[f'(4) = \dfrac{-11}{4}\right] \\ .2 = [f'(-2) = -801] \end{bmatrix}$$

$$Ans3 = \begin{bmatrix} .1 = (f'(x) = 10\,x - 3) & .2 = (f'(x) = 9\,x^2) \\ .3 = (f'(x) = 30\,x^5 + 20\,x^4 + 20\,x^3 - 1) & .4 = \left(f'(x) = 3\,x^3 - 2\,x^2 + \dfrac{5}{6}\right) \\ .5 = \left(f'(x) = -\dfrac{2}{x^{(3/2)}} + \dfrac{1}{\sqrt{x}} - 3\right) & .6 = \left(f'(x) = -\dfrac{4}{5\,x^{(9/5)}} + \dfrac{4}{5\,x^{(1/5)}} + \dfrac{1}{5\,x^{(4/5)}} - \dfrac{5}{4\,x^{(9/4)}}\right) \\ .7 = (f'(x) = -28\,x + 45\,x^2 - 5) & .8 = \left(f'(x) = \dfrac{3\,x + 2\sqrt{x} + 5}{2\sqrt{x}}\right) \\ .9 = \left(f'(x) = -\dfrac{25}{(4\,x - 3)^2}\right) & .10 = \left(f'(x) = \dfrac{45\,x^4 - 1}{2\sqrt{x}}\right) \\ .11 = \left(f'(x) = \dfrac{8\,x^6 - 6\,x^4 - 60\,x^2 - 75}{x^6}\right) & .12 = \left(f'(x) = \dfrac{16\,x^5 + 43\,x^4 - 2\,x^3 - 30\,x^2}{(x+2)^2}\right) \end{bmatrix}, \quad , \quad \begin{bmatrix} \dfrac{:)}{:(} \\ \left[\begin{bmatrix} P \\ V \\ S \\ S \end{bmatrix}\right] \\ \& \\ \left[\begin{bmatrix} M \\ a \\ t \\ h \\ @ \\ M \\ U \\ T \end{bmatrix}\right] \\ \dfrac{:)}{:(} \end{bmatrix}$$

:
:
:
:
: