แบบฝึกหัดเรื่อง เมทริกซ์

ชื่อ-นามสกุล ....

เลขประจำตัว No.2

Determinant

กำหนดเมทริกซ์ $\mathbf{A}-\mathbf{F}$ และค่าคงที่ m , p จงหา

1. $\det(\mathbf{A}) = |-11| = \boxed{-11}$

2. $\det(\mathbf{B}) = |13| = \boxed{13}$

3. $\det(\mathbf{C}) = \begin{vmatrix} -2 & -3 \\ 4 & 4 \end{vmatrix} = (-2)(4)-4(-3) = \boxed{4}$

4. $\det(\mathbf{D}) = \begin{vmatrix} -2 & -12 \\ 10 & 12 \end{vmatrix} = (-2)(12)-10(-12) = -24+120 = \boxed{96}$

5. $\det(\mathbf{E}) = \begin{vmatrix} 4 & 3 & 0 \\ -4 & -3 & 3 \\ 2 & -4 & 2 \end{vmatrix}\begin{matrix} 4 & 3 \\ -4 & -3 \\ 2 & -4 \end{matrix}$

$0 \quad -48 \quad -24$

$-24 \quad 18 \quad 0$

$= (-24+18)-(-48-24) = \boxed{66}$

6. $\det(\mathbf{F}) = \begin{vmatrix} -3 & -4 & -4 \\ -2 & -3 & 4 \\ 4 & 0 & -4 \end{vmatrix}\begin{matrix} -3 & -4 \\ -2 & -3 \\ 4 & 0 \end{matrix}$

$+48 \quad 0 \quad -32$

$-36 \quad -64 \quad 0$

$= (-36-64)-(48-32) = \boxed{-116}$

7. $m\det(\mathbf{C}) - \det(p\mathbf{E}) =$ $m=3, p=4$

$= 3(4) - 4^3(66)$; $\det(pE) = p^n\det(E)$ $(n=3)$

$= \boxed{-4212}$

8. $\det(m\mathbf{D}) - p\det(\mathbf{F}) =$

$= 3^2\det(D) - 4\det(F)$; $\det(mD) = m^n\det(D)$ $(n=2)$

$= 3^2(96) - 4(-116)$

$= \boxed{1328}$

จงแก้สมการต่อไปนี้

9. $\det\left(\begin{bmatrix} x & -4 \\ 4 & 4 \end{bmatrix}\right) = 0$ จงหาค่า x

$4x+16 = 0 \Rightarrow 4x = -16$

$\Rightarrow \boxed{x = -4}$

10. $\det\left(\begin{bmatrix} -3 & 2 & 2 \\ y & -4 & 4 \\ -2 & -3 & 3 \end{bmatrix}\right) = -68$ จงหาค่า y

$(20-6y) - (52+6y) = -68$

$-32-12y = -68 \Rightarrow y = \boxed{3}$

Inverse

11. กำหนดเมทริกซ์ $\mathbf{G}$ จงแสดงการหา $\mathbf{G}^{-1}$

$G = \begin{bmatrix} 3 & 5 \\ 5 & 4 \end{bmatrix}$ (a, b, c, d)

$\det(G) = 12-25 = -13 \neq 0$

หา G^{-1} ได้

$G^{-1} = \frac{1}{\det(G)} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{-13}\begin{bmatrix} 4 & -5 \\ -5 & 3 \end{bmatrix}$

$\boxed{G^{-1} = \begin{bmatrix} \frac{-4}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{-3}{13} \end{bmatrix}}$

Cramer's rule

จงใช้กฎของคราเมอร์หาผลเฉลยของระบบสมการต่อไปนี้

12. System1 = $\begin{cases} -3x+y = 25 \\ 2x+5y = 6 \end{cases}$

ได้ $A=\begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix}$, $\det(A) = (-3)(5)-2(1) = \boxed{-17} \neq 0$

ใช้กฎของคราเมอร์ได้

$$x = \frac{\det(A_1)}{\det(A)} = \frac{\begin{vmatrix} 25 & 1 \\ 6 & 5 \end{vmatrix}}{(-17)} = \frac{25(5)-6(1)}{(-17)} = \boxed{-7}$$

$$y = \frac{\det(A_2)}{\det(A)} = \frac{\begin{vmatrix} -3 & 25 \\ 2 & 6 \end{vmatrix}}{(-17)} = \frac{(-3)(6)-2(25)}{(-17)} = \boxed{4}$$

ดังนั้น $x=-7$ และ $y=4$

13. System2 = $\begin{cases} -4x+y = 16 \\ -5x-4y = 41 \end{cases}$

ได้ $A=\begin{bmatrix} -4 & 1 \\ -5 & -4 \end{bmatrix}$, จะได้ $\det(A) = (-4)(-4)-(-5)(1) = \boxed{21} \neq 0$

$$x = \frac{\det(A_1)}{\det(A)} = \frac{\begin{vmatrix} 16 & 1 \\ 41 & -4 \end{vmatrix}}{21} = \frac{16(-4)-41(1)}{21} = \boxed{-5}$$

$$y = \frac{\det(A_2)}{\det(A)} = \frac{\begin{vmatrix} -4 & 16 \\ -5 & 41 \end{vmatrix}}{21} = \frac{(-4)(41)-(-5)(16)}{21} = \boxed{-4}$$

ดังนั้น $x=-5$ และ $y=-4$

14. System3 = $\begin{cases} 3x+2z = -8 \\ 2x-2y = -8 \\ -2y-2z = 2 \end{cases}$

ได้ $A=\begin{bmatrix} 3 & 0 & 2 \\ 2 & -2 & 0 \\ 0 & -2 & -2 \end{bmatrix}$, จะได้ $\det(A) = \begin{vmatrix} 3 & 0 & 2 \\ 2 & -2 & 0 \\ 0 & -2 & -2 \end{vmatrix} = \boxed{4}$

$$x = \frac{\det(A_1)}{\det(A)} = \frac{\begin{vmatrix} -8 & 0 & 2 \\ -8 & -2 & 0 \\ 2 & -2 & -2 \end{vmatrix}}{4} = \frac{((-32)+32)-(-8)}{4} = \boxed{2}$$

$$y = \frac{\det(A_2)}{\det(A)} = \frac{\begin{vmatrix} 3 & -8 & 2 \\ 2 & -8 & 0 \\ 0 & 2 & -2 \end{vmatrix}}{4} = \frac{(48+8)-32}{4} = \boxed{6}$$

$$z = \frac{\det(A_3)}{\det(A)} = \frac{\begin{vmatrix} 3 & 0 & -8 \\ 2 & -2 & -8 \\ 0 & -2 & 2 \end{vmatrix}}{4} = \frac{(-12+32)-48}{4} = \boxed{-7}$$

ดังนั้น $x=2$, $y=6$, $z=-7$

15. System4 = $\begin{cases} 3y-3z = 9 \\ -x+3z = 10 \\ x+2y = 16 \end{cases}$

ได้ $A=\begin{bmatrix} 0 & 3 & -3 \\ -1 & 0 & 3 \\ 1 & 2 & 0 \end{bmatrix}$, จะได้ $\det(A) = \begin{vmatrix} 0 & 3 & -3 \\ -1 & 0 & 3 \\ 1 & 2 & 0 \end{vmatrix} = \boxed{15} \neq 0$

$$x = \frac{\det(A_1)}{\det(A)} = \frac{\begin{vmatrix} 9 & 3 & -3 \\ 10 & 0 & 3 \\ 16 & 2 & 0 \end{vmatrix}}{15} = \frac{(144-60)-54}{15} = \boxed{2}$$

$$y = \frac{\det(A_2)}{\det(A)} = \frac{\begin{vmatrix} 0 & 9 & -3 \\ -1 & 10 & 3 \\ 1 & 16 & 0 \end{vmatrix}}{15} = \frac{(27+48)-(-30)}{15} = \boxed{7}$$

$$z = \frac{\det(A_3)}{\det(A)} = \frac{\begin{vmatrix} 0 & 3 & 9 \\ -1 & 0 & 10 \\ 1 & 2 & 16 \end{vmatrix}}{15} = \frac{(30-18)-(-48)}{15} = \boxed{4}$$

ดังนั้น $x=2$, $y=7$, $z=4$

Matrices02 for No.2

$$A=[-11],\ B=[\ 13],\ C=\begin{bmatrix}-2 & -3\\ 4 & 4\end{bmatrix},\ \mathrm{D}=\begin{bmatrix}-2 & -12\\ 10 & 12\end{bmatrix},\ E=\begin{bmatrix}4 & 3 & 0\\ -4 & -3 & 3\\ 2 & -4 & 2\end{bmatrix},\ F=\begin{bmatrix}-3 & -4 & -4\\ -2 & -3 & 4\\ 4 & 0 & -4\end{bmatrix},\ \begin{bmatrix}m=3\\ p=4\end{bmatrix}$$

$$No09=\left[\det\left(\begin{bmatrix}x & -4\\ 4 & 4\end{bmatrix}\right)=0\right],\ No10=\left[\det\left(\begin{bmatrix}-3 & 2 & 2\\ y & -4 & 4\\ -2 & -3 & 3\end{bmatrix}\right)=-68\right],\ No11=\left[G=\begin{bmatrix}3 & 5\\ 5 & 4\end{bmatrix}\right]$$

$$System1=\begin{bmatrix}-3\,x+y=25\\ 2\,x+5\,y=6\end{bmatrix},\quad ,\ System2=\begin{bmatrix}-4\,x+y=16\\ -5\,x-4\,y=41\end{bmatrix},\quad ,\ System3=\begin{bmatrix}3\,x+2\,z=-8\\ 2\,x-2\,y=-8\\ -2\,y-2\,z=2\end{bmatrix},\quad ,\ System4=\begin{bmatrix}3\,y-3\,z=9\\ -x+3\,z=10\\ x+2\,y=16\end{bmatrix}$$

Matrices02 Answers for No.2

ExerciseMatrices02 Answers for No.2

01 : det(A) = -11 02 : det(B) = 13

03 : det(C) = 4 04 : det(D) = 96

05 : det(E) = 66 06 : det(F) = -116

07 : m*det(C)-det(pE) = -4212 08 : det(m*D)-p*det(F) = 1328

09 : x = -4 10 : y = 3

$$No11 = \left(\mathrm{Inv}(G) = \begin{bmatrix} \frac{-4}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{-3}{13} \end{bmatrix} \right), \quad , \begin{bmatrix} \\ \\ \end{bmatrix}$$

12 : det(A) = -17 x = -7 y = 4

13 : det(A) = 21 x = -5 y = -4

14 : det(A) = 4 x = 2 y = 6 z = -7

15 : det(A) = 15 x = 2 y = 7 z = 4